Innovation and Imitation in a Model of North-South Trade

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Abstract

This paper analyzes a model of North-South trade with endogenous R&D processes in both regions. North conducts innovating R&D which results in development of new, improved quality varieties. South is involved only in copying at a certain lag behind the North on the quality (product) scale. The question of interest is the impact of trade and IPR protection on the intensive margin of innovation (size of quality jumps) and the extensive margin of imitation (distance in quality, i.e. quality lag of the South) in a steady-state equilibrium. Furthermore, we attempt to derive the growth and welfare effects of trade and IPR policy and to provide possible answers to some questions brought by globalization. Three alternative scenarios are considered: autarky case as a benchmark, free trade case with information protection, and finally, restricted trade case where we introduce patents of finite length that insure a particular form of international protection. We find that opening to trade increases the growth rate of both regions while it also results in larger lag in the quality level in the South. Furthermore, when comparing the effects of different IPR protection tools in the trading world, we find differences in the magnitude of the effects on the variables of interest and also different direction of the change in the relative purchasing power which may even reverse the change in relative welfare when patent length is used as the IPR protection tool. However, common for both policies is that stronger protection decreases world growth rate and increases the quality lag of the South behind the North.
1 Introduction

The issues of trade and globalization have been popular debate topics in economic literature for more than two decades now. Currently, the major questions are derived from the fear of developed North of being taken-over by their trading partners from developing South in terms of economic dominance and power. When looking from the side of the South, that fear becomes an ambition and calls for appropriate "catching-up" policy regimes. It is often argued that the South enters the race using the means of unfair competition, through imitation and intellectual theft. Therefore, the questions of intellectual property rights (IPR) protection, often violated by developing countries, are being raised simultaneously. Having witnessed the "miracle" of the East Asian Tigers in the previous decade, it comes as natural for one to wonder how the China case will evolve, given the size of this economy and its economic potentials. One might also ask whether this Northern fear of losing economic dominance is indeed reasonable. Numerous papers have already dealt with this issue. However, here we use a different approach to the notion of vertical innovation, incorporating the idea of vintages, and in this framework we apply different modeling of the IPR regime and the target of imitative effort of the South. Thus, the problem might as well deserve another attempt of North-South trade modeling.

This paper addresses the questions raised above through a model of North-South trade which attempts to be in accordance with the empirical evidence on current world trade patterns. Namely, the existing literature on North-South trade assumes that the trade pattern is determined through endowment-driven specialization in different industries, or in horizontally differentiated goods of the same value within the same industry due to copying activity. In case of vertical intra-industry differentiation of goods, industries are modeled as monopolies, so imitation results in a product cycle with the whole industry being shifted between regions. The implied trade pattern is again of the inter-industry type. However, recent empirical evidence suggests that the North-South head on competition for dominance over industries might be exaggerated. Not neglecting the importance of the inter-industry specialization of countries and thus of one-way-trade flows in the
world, Fontagne et al. (2008) presents strong evidence of endowment (technology) driven North-South vertical specialization within industries. North includes USA, Japan and EU25 as developed countries, while South is a group of emerging countries (China, Russia, Brazil, India and other emerging). When traded goods are distinguished according to the unit quality level, specialization in different quality ranges is revealed. It is argued that export bundles of these two regions are very similar at the industry level (low importance of inter-industry specialization), half that similar at the product level (some evidence on horizontal intra-industry specialization), while the export structure is completely different at the varieties (of the same product) level. South exports consist of low quality varieties, while North exports the high quality ones, which grants support for the intra-industry vertical specialization and the two-way trade in varieties within products. In terms of market shares, there is strong evidence of down-market share shifts in favor of South, while in the up-market North has the advantage (mainly due to strong position of EU25). At the same time, within South only China is slightly gaining up-market shares. Thus, there might be some ground for the Northern fear of competition from increasing South. However, this fear should not be exaggerated and based on the common argument that "Large South, led by China, will soon become the manufacturing factory of the world and overtake this and other industries from the North". The argument should be based on the degree of the technological development of the South and it’s incentive and ability to advance in overtaking the up-market shares, no matter what industry in question. Undoubtedly, IPR violation helps South in doing so, but it seems that at least in the case of EU25 North has been able to resist the competitive pressure of the South through specialization in high qualities. Following the empirical findings, the aim of this paper is twofold. On one hand, we attempt to develop a theoretical framework which replicates the new specialization pattern, and on the other, we use it for the analysis of endogenous technological advancement of the North and the South, impact of IPR protection policy and welfare implications.

We develop an endogenous R&D model of two regions and analyze three cases (scenarios) based on the degree of trade openness and IPR policy. North conducts innovating R&D which results in the creation of new varieties, each of a higher
quality than the preceding one. South is involved in copying of Northern products at a certain lag. In the first "autarky" scenario, there is no trade between the regions, but the South is still able to imitate Northern products at a lag. There are no patents, but North can to some degree prevent the leak of information on the blueprints and thus affect the difficulty of copying. In the second scenario, both regions are open to trade and the difficulty of copying is still affected by the protection of information. In the third scenario, regions are open but the trade is restricted. Namely, although North can hardly prevent South from copying, it can ensure that Southern copies of protected varieties are not placed in the Northern markets. In this scenario, trade occurs only in goods which are produced exclusively by one country; goods whose patent expired are made and traded by the South, while high-tech goods which are too advanced to be copied are produced and traded by the North. The middle range varieties are not traded. This paper analyzes the determinants of the innovation effort in the North and the distance (on the quality-product scale) between the highest quality goods in the North and in the South in a steady-state equilibrium within the three stated set-ups. The question of the incentives and the mechanisms for closing the gap is implicitly addressed, as well.

1.1 Literature

The foundation for this study comes from two related groups of literature - great body of literature on trade and growth (particularly in the North-South framework) which will not be addressed here, and the literature on product cycles with the issues of innovation and imitation. Vernon (1966) was first to raise the question of the North-South shift of production location in different stages of a product’s life, but like in its modern version presented by Antras (2005), technological transfer comes as a result of optimizing behavior of Northern firms. Following the initial steps of Krugman (1979), Grossman and Helpman (1991a,b) are the most influential articles in which production shifts occur due to imitation in an endogenous growth set-up. In both articles, North is endowed with a comparative advantage, high enough to ensure that innovation takes place in the North, and the production transfers to the South due to endogenous imitation whose intensity determines
the time of the transfer. They combine the notions of quality ladder or variety expansion with product cycle to develop a theoretical framework for analyzing the simultaneous behavior of innovation and imitation rates. Although our paper relies on that framework in many aspects, one of the main differences is in the target of copying effort. Initially, South does not aim at imitating state-of-the-art products, but starts with reproducing less advanced goods. In other words, the endogenous level of R&D effort in the South determines how far in terms of quality level the imitation can reach, and not how fast and in what share it can replicate the most advanced industries of the North.

In a different set-up, Horowitz and Lai (1996) consider a technological lag in the sense that imitators can not copy the most advanced products; however, this lag is a result of legal binding through the patents of a certain length, and not a consequence of imitators’ optimization given the copying technology. They analyze the case of one country in autarky, so it does not apply closely to the problem here, but nevertheless raises an important issue of patent policy (patent length) and its impact on both the size and the frequency of innovation. Among numerous other articles, Segerstrom et al. (1990) also addresses patent policy in terms of the patent length. It uses a set-up where North innovates and the production shifts to the South due to lower costs, but again only after a certain patent expires. However, most papers introduce only a certain policy parameter measuring the level of protection in the sense of how hard (legally or technically) it is to copy a product successfully, but once that occurs, a copy competes freely with the original in the market, and normally wins the battle due to lower costs. This parameter is also present in all our scenarios and represents the only tool of protection policy in the first two set-ups. The additional policy tool is introduced in the third scenario in the form of patent of finite length which at the same time determines the measure of qualities produced in the North. Having in mind the bad experience of international community in their fight both for local IPR protection in developing world and also for international IPR enforcement, we do not assume that North can prevent their protected inventions from being copied. It can only successfully ban them from their internal, Northern market.

The framework in which this paper deals with the issues of the North-South
trade mostly resembles Segerstrom and Dinopoulos (2006) whose structure of presentation is followed closely, as well. They study a quality ladder model with fixed number of industries, each having only state-of-the-art goods produced and consumed, given the form of the utility function. Once it is copied, a variety’s production moves to the South, but returns to the North when that specific industry successfully innovates again. In our paper, varieties increase, but a notion of vintages is used to enrich the variety-expanding set-up with the improvements in quality. In this way, it is allowed for the production and consumption of the whole range of qualities, not only state-of-the-art, which are being copied one after the other up to a certain level on that quality-product scale.

The rest of the paper is organized as follows: section 2 presents the three set-ups of the model (autarky, free trade and restricted trade) and solves for the steady-state equilibrium, section 3 presents the numerical exercise, while section 4 concludes and suggest the following steps of this research.
2 The Model

2.1 Autarky

2.1.1 Overview

The model considers two regions, North and South, which differ in the abilities to conduct R&D and in the wages \((w)\) their workers earn, with Northern wage being higher than the one in the South \((w_N(t) > w_S(t))\). As presented in the figure below, there is a continuum of goods in the world market indexed by \(z(t) \in [-\infty, n_N(t)]\) and each good is characterized by higher quality than the preceding one. Innovation is conducted by the North and each successful innovation results in a new variety with increased quality compared to the previous one invented. North produces the whole range of existing varieties, \([-\infty, n_N(t)]\), where \(n_N(t)\) grows through innovation. Workers in the South conduct imitative R&D and the highest quality variety copied by the South is \(n_S(t)\), being inside the range of goods produced in the North. The distance between the highest qualities produced in the North and the South is of measure \(d\), i.e. \(d(t) = n_N(t) - n_S(t)\). Thus, the production in the two regions overlaps up to variety \(n_N(t) - d\), while varieties \([n_N(t) - d, n_N(t)]\) have not been copied and are produced only by the North. As there is no trade, the consumption bundles of the two regions are different and consist only of the varieties produced at home.

\[
\begin{array}{c}
-\infty \quad \text{n}_N \\
-\infty \quad \text{d}
\end{array}
\]
2.1.2 Consumers

Population in both regions is fixed and it is of measure $L_N$ and $L_S$ in the North and South, respectively. Each individual supplies one unit of labor inelastically and earns the wage ($w$) which is the same in both manufacturing and R&D, $w_N$ in the North and $w_S$ in the South. Labor is not mobile between the regions.

Consumers in both regions have the same preferences and they maximize lifetime utility of the following form

$$U = \int_{0}^{\infty} e^{-\rho t} \ln u(t) dt,$$

with $\rho > 0$ being the discount factor and $u(t)$ the instantaneous utility given by

$$u(t) = \left\{ \int_{-\infty}^{\gamma(t)} \left( x(z, t)^{1-\alpha} \right)^{\frac{1}{\alpha}} dz \right\}^{\frac{1}{\alpha}} = \left\{ \int_{-\infty}^{\gamma(t)} x(z, t)^{\alpha} dz \right\}^{\frac{1}{\alpha}}.$$

Utility at time $t$ is a quality-augmented CES consumption index with $x(z, t)$ being the consumption of variety $z$ of quality index $e^{\gamma(z)(1-\alpha)}$, where $\gamma$ measures the size of quality improvement of each successive variety and is equal for both North and South. $\alpha$ is a parameter indicating the measure of substitution (with $\frac{1}{1-\alpha}$ as the elasticity of substitution) between different varieties and with $0 < \alpha < 1$ it follows that consumers prefer goods of higher quality (higher $z$).

To derive the optimal demand for each variety consumed given prices, consumers maximize instantaneous utility subject to their individual expenditure on all goods ($C(t)$). This is a problem of static optimization across varieties

$$\max \left\{ \int_{-\infty}^{\gamma(t)} e^{\gamma(z)(1-\alpha)} x(z, t)^{\alpha} dz \right\}^{\frac{1}{\alpha}} \quad \text{subject to} \quad C(t) = \int_{-\infty}^{\gamma(t)} p(z, t)x(z, t) dz,$$

which gives
\[ x(z, t) = \frac{(p(z, t)/e^{\gamma z(1-a)})^{1-\alpha}}{\int_{-\infty}^{\alpha(t)}(p(z, t)^{\alpha}/e^{\gamma z(1-a)})^{1-\alpha}dz} C(t) = p(z, t)^{1-\alpha}e^{\gamma z} \frac{C(t)}{P(t)^{1-\alpha}}. \] (3)

The demand function takes the familiar form, where the share of each variety in the total consumption is given by the share of its quality-price ratio in the index of price-quality ratios of all varieties consumed \((P)\).

With equal price of all varieties in a region (to be proved later), demand functions for each variety in the North and in the South, respectively, can be simplified to

\[ x_N(z, t) = \frac{1}{p_N(t)} \gamma e^{\gamma(z-n_N(t))} C_N(t) \] (4)
\[ x_S(z, t) = \frac{1}{p_S(t)} \gamma e^{\gamma(z-n_N(t)+d)} C_S(t). \] (5)

Dynamic optimization of the lifetime utility given (2), (3) and the budget constraint

\[ \dot{A}(t) = w(t) - C(t) + r(t)A(t), \] (6)

where \(A(t)\) represents individual assets and \(r(t)\) the market interest rate at time \(t\), results in the Euler condition

\[ \frac{\dot{C}(t)}{C(t)} = r(t) - \rho. \] (7)

Expenditure grows only when the market interest rate exceeds subjective discount factor. This paper will analyze a steady-state equilibrium in which wages \((w_N \text{ and } w_S)\) and expenditures \((C_N \text{ and } C_S)\) do not change over time, and thus,
market interest rate will also be constant and equal to the subjective discount factor.

2.1.3 Production

Northern firms conduct innovative R&D which requires certain cost depending on the labor employed in research and the productivity of the process. When new variety is invented, the producer becomes a monopoly. This is due to the fact that under Bertrand competition, no other Northern firm will have an incentive to copy at any time since its entry into the market with the first successful innovator would drive profits down to zero and would not allow for covering the R&D costs of imitation. For this reason, patents are not needed to protect IPR domestically, but will be introduced in the restricted trade scenario as a particular form of international protection. Each good requires only one unit of labor for its production, so the firm faces marginal costs equal to wage \( w_N \). The monopolist determines the product price by maximizing profits subject to the consumers demand,

\[
\max p(z, t)x(z, t) - wx(z, t) \quad \text{subject to } (4), \quad \text{yielding the optimal monopoly price}
\]

\[
p_N(z, t) = p_N = \frac{1}{\alpha}w_N, \tag{8}
\]

which is constant and equal across varieties. This implies that consumers demand across varieties increases with quality level, but demand for each variety decrease over time due to invention of higher quality ones.

Southern firms involve into imitative R&D and incur costs depending on the R&D labor and the productivity of copying, which will also affect the highest quality level copied. When a variety is successfully copied, the imitator becomes a monopolistic producer using one unit of labor per good (marginal costs equal to wage \( w_S \)). The firm charges monopoly price derived as for the North which is needed to compensate it for the R&D costs.
\[ p_S(z, t) = p_S = \frac{1}{\alpha} w_S. \]  \hspace{1cm} (9)

As in the North, under Bertrand competition, no other Southern firm will have an incentive to copy an already copied product since its entry drive profits down to zero and does not allow for covering the R&D costs of copying a copy.

Both regions firms earn profits only at the local markets, and the profits are given by

\[
\begin{align*}
\Pi_N(z, t) &= p_N x_N(z, t) L_N - w_N x_N(z, t) L_N = (1 - \alpha) \gamma e^{\gamma (z - a_N(t))} C_N L_N \quad (10) \\
\Pi_S(z, t) &= p_S x_S(z, t) L_S - w_S x_S(z, t) L_S = (1 - \alpha) \gamma e^{\gamma (z - a_N + d)} C_S L_S. \quad (11)
\end{align*}
\]

At any time \( t \), the innovator’s and imitator’s profits increase in total expenditure \((C_N L_N, C_S L_S)\) and quality jump \((\gamma)\), but they decrease over time as the quality level of the particular variety decreases relative to the growing highest quality produced.

### 2.1.4 R&D Processes

The North employs labor of measure \( R_N \) in research which, if successful, results in the invention of a new good of higher quality. The innovation is characterized by a certain difficulty parameter \( \beta > 0 \) (with \( \frac{1}{\beta} \) being the productivity of innovation) and the R&D process is modeled as

\[
\tilde{\gamma}_N = \frac{R_N}{\beta}. \hspace{1cm} (12)
\]

\( \tilde{\gamma}_N \), as the effective research labor, represents the growth rate of the quality index in the North,
\[ e^{\gamma(1-\alpha)z^N} = e^{\gamma(1-\alpha)z^N_0} e^{\gamma N t}. \] (13)

The above specification implies that \( \gamma(1-\alpha)z^N = \gamma(1-\alpha)z^N_0 + \gamma N t, \) and taking derivative with respect to time, one can obtain that \( \dot{\gamma}(1-\alpha)z^N + \gamma(1-\alpha)\dot{z}^N = \dot{\gamma}(1-\alpha)z^N_0 + \frac{d\gamma}{dt} z^N + \dot{\gamma} N t. \) From the left-hand side, one can see that technological progress comes in two forms, invention of new goods and increase in quality, which we might call extensive and intensive margins of change, respectively. Since it would be too cumbersome to analyze both as endogenous simultaneously, one margin should be pinned down. Unlike the usual growth literature practice of taking the size of each quality jump as constant and exogenous, and then analyzing the endogenous innovation rate, here we assume exactly the opposite. The invention frequency is exogenous and new products arrive along with time, i.e. \( \dot{z}^N = 1, \) but the size of quality improvement with each new product is left free to be determined endogenously. In this way, the ranges of varieties can be also regarded as the measure of time, so that \( d \) in fact represents the lag of South in time. The analysis will focus on the balanced growth path (BGP) with constant growth rate of the quality index \( \big( \frac{d\gamma}{dt} N = 0 \big) \) and the constant size of endogenous quality jumps \( (\dot{\gamma} = 0) \), so that the last expression collapses to

\[ \gamma(1-\alpha)\dot{z}^N = \gamma(1-\alpha) = \dot{\gamma} N, \quad \text{with} \quad \dot{z}^N = 1. \]

Imitation is conducted by Southern R&D labor of measure \( R_S \) with the difficulty parameter \( \theta(d) > 0, \) which is proportional to \( \beta, \) but depends on the North-South distance. Namely, it can be argued that as South attempts to imitate more intensively and decrease the quality gap relative to the North, the copying process increases in difficulty, and thus, \( \theta \) is assumed to be a decreasing function of \( d. \) This factor of proportionality to \( \beta \) is given by the ratio of highest quality in the South and the one in the North. An additional parameter, \( \eta, \) represents the degree of information protection by the North and directly affects the difficulty (productivity) of copying So, the productivity of copying \( (\frac{1}{\beta}) \) is decreasing in \( \eta \) and increasing in \( d, \) and with free flow of information \( (\eta = 1) \) and no distance in quality \( (d = 0), \) \( \theta \) becomes equal to \( \beta. \)
\[
\theta(d) = \eta \beta \frac{e^{\gamma(n_N - d)(1-\alpha)}}{e^{\gamma n_N(1-\alpha)}} = \eta \beta e^{-\gamma d(1-\alpha)}. \quad (14)
\]

The effective Southern research labor, \(\hat{\gamma}_S\), gives the growth rate of the Southern quality index.

\[
\frac{R_S}{\theta(d)} = \hat{\gamma}_S \\
\exp(\gamma(1-\alpha)z^S) = \exp(\gamma(1-\alpha)z^S \hat{\gamma}^S) \quad (15)
\]

As for the North, \(\hat{\gamma} = 0\) and \(\frac{dz^N}{dt} = 0\) at the BGP, so that here also \(\gamma(1-\alpha)z^N = \hat{\gamma}^N\). The main trade-off in this paper will be expressed in the relation between the intensive margin of innovation (given by \(\gamma\)) and the imitation threat coming from the South (given by \(d\)), in a steady-state equilibrium. To analyze this relation, we look for a steady-state equilibrium in which quality (or product) distance \(d = n_N - n_S\) between North and South is constant, implying that for each new product invented, one new product is copied. In other words, \(z\) is the same in both regions, so it equals 1 in the South as well. Finally, this implies that \(\hat{\gamma} = \gamma(1-\alpha)\) and equal in both regions.

2.1.5 R&D Optimization

The expected benefit of a successful R&D effort (value of new variety) is represented by expected discounted profits from innovating or copying in the North and the South, respectively. Having assumed that derivative of quality index with respect to time \(\frac{dz}{dt} = \hat{z} = 1\), it is convenient in computational sense to discount the profit flows over the varieties \(z\), since under given assumptions it is equivalent to discounting over time.

With wages, prices and expenditures constant over time, profits change due to the growth in the price index. Therefore, the values of a new variety \((V_N)\) and a copy \((V_S)\) are given by
\[ V_N = (1 - \alpha) \gamma e^{\gamma n_N} C_N L_N \int_{n_N}^{\infty} e^{-\gamma z} e^{-r(z-n_N)} dz = (1 - \alpha) \frac{\gamma}{\gamma + r} C_N L_N \quad (17) \]

\[ V_S = (1 - \alpha) \gamma e^{\gamma (n_N - d)} C_S L_S \int_{n_N - d}^{\infty} e^{-\gamma z} e^{-r(z-n_N - d)} dz = (1 - \alpha) \frac{\gamma}{\gamma + r} C_S L \quad (18) \]

The value of introducing a new variety is increasing in total consumer expenditure and size of the quality jump, while it is decreasing in the elasticity of substitution between varieties.

The entry into the R&D races is free and all participants have access to the same R&D technology, so the benefits of winning a race will equal the costs of R&D in a steady-state equilibrium. Since current monopolists already incurred R&D costs and are earning monopoly profits, free entry into R&D races implies that there is no incentive for them to participate until continuing their current production becomes non-optimal.

Given the formulation of R&D technology, research labor required for each innovation \((z = 1)\) in the North is given by

\[ R_N = \beta \gamma (1 - \alpha), \quad (19) \]

and with \(w_N\) as the cost of each unit of research labor, the optimal R&D condition (arbitrage condition) in the North is given by

\[ V_N = w_N \beta \gamma (1 - \alpha) \quad (20) \]

Combining this condition with the expression for \(V_N\), it yields

\[ \frac{1}{\gamma + r} C_N L_N = w_N \beta. \quad (21) \]

Similar derivation applies also to the South. Research labor needed for one new copy is given by
\[ R_S = \theta(d)\gamma(1 - \alpha), \quad (22) \]

which, with wage \( w_S \), yields the arbitrage condition

\[ \frac{1}{\gamma + r}C_SL_S = w_S\theta(d). \quad (23) \]

### 2.1.6 Labor Markets

Full employment of labor requires that in both regions at any time \( t \) all workers are employed in either R&D or manufacturing. Under the assumption of \( dz = dt \), at each point in time, total R&D labor in either region is actually equal to the labor requirement for the development of one new product or a copy given by (19) for the North and (22) for the South. Therefore, the full employment labor market conditions for the two regions are given by

\[ L_N = R_N + \int_{-\infty}^{n_N} D_N L_N dz = \beta\gamma(1 - \alpha) + \frac{C_N L_N}{p_N} \quad (24) \]

\[ L_S = R_S + \int_{-\infty}^{n_{N-d}} D_S L_S dz = \theta(d)\gamma(1 - \alpha) + \frac{C_S L_S}{p_S} \quad (25) \]

### 2.1.7 Steady-State Equilibrium Analysis

Combining the full employment conditions (24) and (25) with the R&D optimization conditions given by (21) and (23) for the North and the South, respectively, two steady-state equilibrium conditions are obtained,

\[ L_N = \beta\gamma(1 - \alpha) + \frac{w_N\beta}{p_N}(\gamma + r) = \beta(\gamma + r\alpha) \quad (26) \]

\[ L_S = \theta(d)\gamma(1 - \alpha) + \frac{w_S\theta(d)}{p_S}(\gamma + r) = \theta(d)(\gamma + r\alpha). \quad (27) \]
The Northern condition determines the endogenous size of the quality jump \((\gamma)\) as

\[
\gamma = \frac{L_N}{\beta} - r\alpha. \tag{28}
\]

The quality jump depends positively on the productivity of R&D labor \(\left(\frac{1}{\beta}\right)\), while bigger interest rate (equal to subjective discount factor) and bigger \(\alpha\) (higher elasticity of substitution) decrease \(\gamma\) due to their negative impact on the value of the innovation.

The Southern equilibrium condition determines the quality lag \((d)\) as a function of \(\gamma\),

\[
d = \frac{1}{1 - \alpha} \ln\left(\frac{L_N}{L_S} \eta\right) \frac{1}{\gamma}. \tag{29}
\]

Higher \(\gamma\) implies smaller distance between North and South as it increases the value of imitation and R&D labor productivity, which together counteracts the increase in the R&D labor cost. Elasticity of substitution and degree of information protection both increase the distance as they decrease the value of copying and the productivity of imitative R&D labor, respectively.

Consumers in both regions maximize their lifetime utility subject to budget constraint given by the expression for the change in assets they possess \((A)\), \(\dot{A} = w - C + rA\). It is assumed that each region’s consumers own only their local companies, and not those of the other country. Since the growth rate of assets, defined as \(\frac{\dot{A}}{A} = \frac{w-C}{A} + r\), will be constant in a steady-state equilibrium with constant \(w, C\) and \(r, A\) will also be constant. This implies that \(C-w = rA\), and in aggregate form

\[
C_N L_N = w_N L_N + r\bar{A}_N \quad \text{for the North, and}
\]

\[
C_S L_S = w_S L_S + r\bar{A}_S \quad \text{for the South.}
\]
$\bar{A}_N$ represent total Northern assets which are equal to the sum of the values of all existing firms in the North at time $t$, while $\bar{A}_S$ stands for the total assets in the South, equal to the sum of the values of all copies at a given time $t$. Therefore, $\bar{A} = \int_0^\infty V(a) da$, where $V(a)$ is the value of a periods old firm at time $t$. This yields the expenditure conditions

$$
C_N L_N = (1 - \alpha) r w_N \beta + w_N L_N \quad (30)
$$

$$
C_S L_S = (1 - \alpha) r w_S \theta(d) + w_S L_S \quad (31)
$$

Utility in both region is equal to $\frac{C}{P}$, and with constant consumer expenditure, utility growth is given by the negative of the growth of quality-price index

$$
\frac{\dot{u}}{u} = \frac{1 - \alpha}{\alpha} \frac{de^{\gamma n}}{dt} \frac{1}{e^{\gamma n}} = \frac{1 - \alpha}{\gamma} = (1 - \alpha) \left( \frac{L_N}{\alpha \beta} - r \right) \quad (32)
$$
2.2 Free Trade

The model now considers two regions, North and South, which are open to trade. South still produces varieties up to the one at distance $d$ from the highest quality variety in the North. Since $w_N > w_S$, South can produce these varieties at a lower cost and due to free trade, it is no longer optimal for the North to continue their production. However, the range of varieties that have not been copied by the South, $[n_N(t) - d, n_N(t)]$, are produced and traded exclusively by the North. As presented in the figure below, there is a continuum of goods in the world market indexed by $z(t) \in [-\infty, n_N(t)]$, but there is no overlapping in the production as in the autarky case; South specialize in the production and trade of low quality varieties, while North specializes in the high quality ones. IPR protection policy is still represented only by the degree of information protection which affects the difficulty of copying and there are no patents to protect innovation internationally.

The composition of the consumption bundles is the same in both regions as now Southern consumers have access to the whole range of varieties due to trade. Since all world consumers are buying a particular variety at the same price (markup over the marginal cost in the region of production which is unique due to no overlapping), the quality-price index is the same in both regions.

$$P_N = P_S = \left\{ \frac{1}{\gamma} e^{\gamma n_N} \left[ \frac{n}{p_N} (1 - e^{-\gamma d}) + \frac{n}{p_S} e^{-\gamma d} \right] \right\}^{\frac{\sigma - 1}{\sigma}}$$ (33)
Northern (Southern) firms conduct innovative (imitative) R&D and after a new variety is invented, its production requires one unit of labor for each unit of the good. The monopolist (innovator or imitator) determines the product price by maximizing profits subject to the consumers demand which again, yields the optimal monopoly price

\[ p_i(z, t) = p_i = \frac{1}{\alpha} w_i, \quad i = N, S \]  

(34)

However, the revenue now comes from both domestic and foreign market, and for North it is on the \([n_N(t) - d, n_N(t)]\) range of varieties, while South sells varieties in the range \([-\infty, n_N(t) - d]\). Value of a new variety or a new copy is still determined as discounted stream of profits (now from both home and foreign market) over the period of firm’s operation. However, the life of a variety in the North is now not infinite but terminates at the time it is successfully copied by the North, i.e. \(d\) periods after the invention. Therefore, the time span over which profits are discounted is now different in the North and the South, and the values of innovation and imitation, respectively, are given by

\[ V_N = \frac{1 - \alpha}{\alpha} w_N p_N \frac{1}{\gamma} \frac{1}{r + \gamma} \frac{C L}{\tilde{P}} (1 - e^{-\gamma d}) \]  

(35)

\[ V_S = \frac{1 - \alpha}{\alpha} w_S p_S \frac{1}{\gamma} \frac{1}{r + \gamma} \frac{C L}{\tilde{P}} e^{-\gamma d}; \]  

(36)

where \(\tilde{P} = P \frac{\alpha}{\gamma} e^{-\gamma n_N}\) and \(C L = C_N L_N + C_S L_S\). R&D technology is defined in the same way as in the autarky scenario, and the arbitrage conditions for the North and the South, obtained by equalizing the benefits and costs of R&D, are given by

\[ \frac{1 - \alpha}{\alpha} \frac{1}{p_N^\gamma} \frac{1}{r + \gamma} \frac{C L}{\tilde{P}} (1 - e^{-\gamma d}) = \beta \gamma \]  

(37)

\[ \frac{1 - \alpha}{\alpha} \frac{1}{p_S^\gamma} \frac{1}{r + \gamma} \frac{C L}{\tilde{P}} e^{-\gamma d} = \theta(d) \gamma \]  

(38)
The full employment labor market conditions for the two regions are given by

\[ L_N = \beta \gamma (1 - \alpha) + p_N^{1 - \frac{1}{\gamma}} \frac{1}{\gamma} \frac{CL}{P} (1 - e^{-\gamma d}) \]  
(39)

\[ L_S = \theta(d) \gamma (1 - \alpha) + p_S^{1 - \frac{1}{\gamma}} \frac{1}{\gamma} \frac{CL}{P} e^{-\gamma d}, \]  
(40)

which, when combined with the arbitrage conditions in the North and the South, (37) and (38), yield the first two steady-state equilibrium conditions, both endogenous in \( \gamma \) and \( d \).

\[ L_N = \beta \gamma (1 - \alpha) + \alpha \beta (\gamma + r) \frac{1 - e^{-\gamma d}}{1 - e^{-(\gamma + r)d}} \]  
(41)

\[ L_S = \theta(d)(\gamma + \alpha r) \]  
(42)

In the free trade scenario, the size of quality jumps is not determined exclusively by the North, but also depends on the conditions of the South, so that \( \gamma \) and \( d \) are jointly determined by the two equations above. However, since the term \( \frac{1 - e^{-\gamma d}}{1 - e^{-(\gamma + r)d}} \) is necessarily smaller than 1, it follows that \( \gamma \) in the free trade case is unambiguously larger than \( \gamma \) in the autarky. Compared to autarky scenario, the market for any world producer is now larger as it includes both regions, but since we assume financial autarky, trade is balanced. For South, it implies that the increased range of varieties in the Southern consumption bundle alone has no effect on their producers’ revenue. Also, the life of any variety produced in South is still infinite, so the Southern equilibrium condition is unchanged (though \( \gamma \) and \( d \) in equilibrium will be different). Northern consumption bundle is unchanged when moving from autarky to free trade. However, in the North, varieties now live only \( d \) periods. The value of an innovation is proportional to only \( 1 - e^{-(\gamma + r)d} \) share of total expenditure, while the total demand for Northern production (and thus, manufacturing labor) depends on the \( 1 - e^{-\gamma d} \) share of expenditures. Since the latter is smaller, it follows that there is a decrease in Northern manufacturing
employment which must be accompanied by an increase in the research labor in order to restore full employment. As a result, $\gamma$ is higher.

The effect of increase in the size of the quality jump now have an ambiguous effect on the North-South distance in quality which will depend on the parameters of the model and will be analyzed in the numerical exercise. $d$ in free trade is given by

$$
d = \frac{1}{1 - \alpha} \ln\left(\frac{\eta \beta (\gamma + \alpha r)}{L_S}\right) \frac{1}{\gamma}
$$  \hspace{1cm} (43)

With lower elasticity of substitution between varieties, lower interest rate and more open flow of information ($\eta$ closer to 1), it is more likely that the equilibrium $d$ will be lower compared to autarky. No matter what change in $d$ will occur, the increase in $\gamma$ is unambiguous and the equilibrium $d$ will in turn only affect the magnitude of this increase. It is interesting to note that through this channel, $\eta$ will have an effect on $\gamma$ as well, and thus on the common growth rate in both regions, still given by

$$
\frac{\dot{u}}{u} = \frac{1 - \alpha}{\alpha} \gamma.
$$  \hspace{1cm} (44)

The third endogenous variable in the model is the relative wage ($\omega$) defined as $\frac{w_N}{w_S}$. Dividing Northern arbitrage condition by the one from the South yields the following condition

$$
\omega^{\frac{1}{1-\alpha}} = \frac{\frac{1}{\sigma(d)} e^{-\gamma d}}{\frac{1}{\sigma(d)} e^{-\gamma d}}
$$  \hspace{1cm} (45)

where the relative wage is proportional to the ratio of R&D productivities multiplied by the terms referring to the varieties lifetime over which the profits are discounted in the North and the South. Thus, the relative wage in fact comes from the ratio of factual productivities in creating the value of new businesses in the two regions. When simplified, the relative wage condition determines $\omega$ as
\[ \omega = \left[ (1 - e^{-(\gamma + r)d})^{\gamma} \sigma \right]^{1/\alpha}. \] (46)

Both \( \gamma \) and \( d \) have a positive impact on the relative wage, and so does the degree of information protection which decreases the productivity of copying. For the model to be one of the North-South trade, it is necessary for monopolistic price in the South to be lower than the competitive price in the North (higher price implies limit pricing by the South, but is not analyzed here as it does not affect the generality of the results). Therefore, the equilibrium \( \omega \) has to be at least \( \frac{1}{\alpha} \) which proves to be the case with a wide range of parameters used in the numerical exercise.
2.3 Restricted Trade

In the third, restricted trade scenario patents of a finite length are introduced as a measure of international IPR protection. We do not assume patents can prevent South from imitating, but the copies of still protected originals are banned from the Northern market. Since South produces these varieties at a lower cost, it has no incentive to import them from the North, so that there is an interval of nontraded varieties, those that are protected in the North but already copied by the South. Thus, trade occurs only in the ranges in which the regions are sole producers; the North imports low quality varieties that are produced by the South, \([-\infty, n_N(t) - T]\), while the South imports the high quality range from the North, \([n_N(t) - d, n_N(t)]\).

\[
\begin{align*}
-\infty & \quad \text{traded by South} \quad \text{traded by North} \quad \text{NOT traded} \\
\text{traded} & \quad \text{by South} \quad \text{traded} \quad \text{by North} \quad \text{NOT} \\
T & \quad \text{d}
\end{align*}
\]

The bundles consumed by North and South are of the same composition, however, the prices of varieties in the nontraded range are now different in the two markets. Therefore, the price-quality indices in the North and the South are different, as well, and given by

\[
P_N = \left\{ \frac{1}{\gamma} e^{\gamma n_N} \left[ \frac{\alpha}{p_N} (1 - e^{-\gamma T}) + \frac{\alpha}{p_S} e^{-\gamma T} \right] \right\}^{\frac{\alpha-1}{\alpha}} \quad (47) \\
P_S = \left\{ \frac{1}{\gamma} e^{\gamma n_N} \left[ \frac{\alpha}{p_N} (1 - e^{-\gamma d}) + \frac{\alpha}{p_S} e^{-\gamma d} \right] \right\}^{\frac{\alpha-1}{\alpha}}. \quad (48)
\]
Value of a new variety or copy is determined as discounted stream of profits over the periods of firms’ operation, and it should be noted that the production of a variety in the North does not cease with its successful imitation but only after the patent expires. Once it has been imitated, the variety is no longer shipped to the South who produces it by itself, but it is still placed in the Northern market. Therefore, both Northern and Southern producers do not earn export profits over the whole period of operation, which is taken into account when discounting. The value of innovation and imitation are given by

\[
V_N = \frac{1 - \alpha}{\alpha} w_N p_N^{\frac{1}{\alpha}} \frac{1}{r + \gamma} \left[ \frac{C_N L_N}{P_N} (1 - e^{-(r+\gamma)T}) + \frac{C_S L_S}{P_S} (1 - e^{-(r+\gamma)d}) \right] \tag{49}
\]

\[
V_S = \frac{1 - \alpha}{\alpha} w_S p_S^{\frac{1}{\alpha}} \frac{1}{r + \gamma} \left[ \frac{C_N L_N}{P_N} e^{-r(T-d)} e^{-\gamma T} + \frac{C_S L_S}{P_S} e^{-\gamma d} \right] \tag{50}
\]

with \( \tilde{P}_i = P_i^{\frac{1}{\alpha}} e^{-\gamma_i N_H} \), for \( i = N, S \). Equating the value of a new variety (copy) to the cost of innovation (imitation) yields the arbitrage conditions in the North and the South, respectively

\[
\frac{1}{\alpha} p_N^{\frac{1}{\alpha}} \frac{1}{r + \gamma} \left[ \frac{C_N L_N}{P_N} (1 - e^{-(r+\gamma)T}) + \frac{C_S L_S}{P_S} (1 - e^{-(r+\gamma)d}) \right] = \beta \gamma \tag{51}
\]

\[
\frac{1}{\alpha} p_S^{\frac{1}{\alpha}} \frac{1}{r + \gamma} \left[ \frac{C_N L_N}{P_N} e^{-r(T-d)} e^{-\gamma T} + \frac{C_S L_S}{P_S} e^{-\gamma d} \right] = \theta(d) \gamma \tag{52}
\]

Full labor employment requires that all workers are allocated either to manufacturing or R&D sectors which translates into the following conditions

\[
L_N = \beta \gamma (1 - \alpha) + p_N^{\frac{1}{\alpha}} \frac{1}{\gamma} \left[ \frac{C_N L_N}{P_N} (1 - e^{-\gamma T}) + \frac{C_S L_S}{P_S} (1 - e^{-\gamma d}) \right] \tag{53}
\]

\[
L_S = \theta(d) \gamma (1 - \alpha) + p_S^{\frac{1}{\alpha}} \frac{1}{\gamma} \left[ \frac{C_N L_N}{P_N} e^{-\gamma T} + \frac{C_S L_S}{P_S} e^{-\gamma d} \right] \tag{54}
\]
Assumed financial autarky implies that in every period export revenues of the two regions are the same, thus the trade balance condition has to be satisfied. This condition holds also in the free trade scenario, but due to no nontraded varieties, price-quality index is the same in both regions and the demand any producer is facing is a share of the total world expenditure divided by the common index. Here, price indices differ and the trade balance condition is used to facilitate solving the model. The condition is given as

$$\frac{C_N L_N}{P_N^{\alpha-1}} = \frac{C_S L_S}{P_S^{\alpha-1}} \omega^{\alpha-1} e^\gamma T (1 - e^{-\gamma d})$$

Combining the arbitrage conditions, (51) and (52), with the labor market conditions, (53) and (54), and using the trade balance, two out of three steady-state equilibrium equations are derived. The third equation comes from the arbitrage conditions combined with the trade balance, which completes the equilibrium system of three equations endogenous in $\gamma$, $d$ and $\omega$.

$$L_N = \beta \gamma (1 - \alpha) + \beta \alpha (r + \gamma) \left( \frac{1 - e^{-T}}{1 - e^{-(r+\gamma)T}} + \omega^{\alpha-1} e^{-T} \right)$$

$$L_S = \theta(d) \gamma (1 - \alpha) + \theta(d) \alpha (r + \gamma) \left( \frac{e^{-T} + \omega^{\alpha-1} e^{-T+T}}{e^{-(r+\gamma)T} e^{rd} + \omega^{\alpha-1} e^{-T+T+T}} \right)$$

$$\omega^{\alpha-1} = \left( \frac{1 - e^{-(r+\gamma)T}}{e^{-(r+\gamma)(T-d)} + \omega^{\alpha-1} e^{-T}} \right)^{-\frac{\gamma}{\gamma d}}$$

As in the free trade scenario, equilibrium conditions in the restricted trade case can be compared to those in the autarky. From the first equilibrium equation, since the fraction term is unambiguously smaller than one, we can conclude that manufacturing labor is lower than in the autarky, which implies reallocation of workers towards R&D and increase in the size of quality jumps. However, the magnitude of this change is now affected also by the patent length $T$ and the
equilibrium relative wage, which makes the analytical analysis complicated enough to call for numerical solutions.
3 Numerical Exercise

This section presents preliminary results of the numerical exercise on a not fully calibrated model. We investigate the impact of increasing information protection ($\eta$) and the patent length ($T$, in the restricted trade case) on the size of the quality jump ($\gamma$) and thus the growth rate of the economy, on the North-South distance in quality ($d$), relative wage ($\omega$), relative utility (welfare) and the relative price-quality index. The interest rate is equal to subjective discount factor and taken to be 0.04 (4 percent). To match the empirical evidence on monopolistic mark-ups in the range of 10-40%, $\alpha$ is set at 0.8 which implies 25% mark-up and the elasticity of substitution between varieties of 5. Since the model exhibits scale effects, which should be corrected for, we assume equal size of the North and the South for now in order to abstract from relative size effects. The productivity of R&D in the North is taken to be 1, while we vary $\eta$ from 1 (free information flow) to 1.5.

Figure 1. presents the results in the autarky scenario. Increase in the protection of information by the North and thus, increase in the difficulty of copying in the South, has no effect on the growth rate, but increases the quality lag of the South.

![Figure 1: The effect of increasing $\eta$ on $\gamma$ and $d$, autarky scenario](image-url)
When the two regions open to trade (Figure 2.), $\gamma$ and $d$ are determined simultaneously by the equilibrium conditions from both North and South. One can see from the graphs below that for any level of $\eta$, the size of quality jumps and also the quality lag of the South are both higher compared to the autarky results due to reasons discussed in the previous section. In the free trade scenario, the effect of $\eta$ on the distance also translates into the change in the size of the quality jump. With higher information protection $\gamma$ decreases, while the relative wage and the relative welfare increase. The effect of $\eta$ on the quality lag is positive as in the autarky case, though the distance is not zero even in the case of free information sharing. It might be concluded that trade necessarily brings incentives for specialization of both regions, no matter how weak the IPR protection policy is.

\begin{figure}[h]
\centering
\begin{subfigure}{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{gamma_graph.png}
\caption{Gamma in \%}
\end{subfigure}
\begin{subfigure}{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{distance_graph.png}
\caption{Distance}
\end{subfigure}
\begin{subfigure}{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{relative_wage_graph.png}
\caption{Relative wage}
\end{subfigure}
\begin{subfigure}{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{relative_utility_graph.png}
\caption{Relative utility}
\end{subfigure}
\caption{The effect of increasing $\eta$ on $\gamma$, $d$, $\omega$ and relative utility, free trade scenario}
\end{figure}

Figures 3. and 4. present the results of numerical solutions in the restricted trade scenario and the effects of different IPR policy measures by the North (increase of $\eta$ and increase of patent length $T$, respectively). Compared to the free trade case, the size of quality jumps is now smaller for any level of protection, while the distance, the relative wage and relative utility are higher. Also, with
increase in $\eta$, the size of quality jumps decreases, dampening the growth of the economies, distance increases, and so do the relative wage and relative welfare. However, the change in all the variables is slower compared to the rates in the free trade scenario, due to stronger IPR protection.

More interestingly, the two channels of IPR policy measures (information protection or patent length) have different welfare effects. For a given $T$, an increase in $\eta$ results in the decreasing relative price index (Figure 3.). This is due to the fact that the rise in quality lag does not affect the prices Northern consumers pay for any variety. However, the Southern price index rises as that region’s consumers pay higher (Northern) prices for the varieties that are now too difficult to be copied. Therefore, the decrease in the relative price index, along with the increase in the relative wage and thus relative expenditure, results in the increase in the Northern welfare relative to the welfare of the South.

![Figure 3: The effect of increasing $\eta$ on $\gamma$, $d$, $\omega$ and relative utility and price index, restricted trade scenario](image)

For a given $\eta$, an increase in the patent length brings about an increase in the relative price index, an opposite effect from the one of the previous IPR policy tool.
Rise in the quality distance is pushing up the price index in the South, but longer $T$ on the other hand implies that now North imports less varieties as more copies are banned, and thus pays higher prices for more low quality varieties. It turns out that the latter effect dominates and the relative price index increases, which works in favor of lower relative utility. At lower patent lengths, the increase in the relative wage is still strong enough to secure rising relative utility, but compared to the effect of $\eta$, the rise in the relative utility is now slower. For even larger patent length, $d$ starts falling and the relative wage (expenditure) increase is not sufficient to reverse now even stronger effect of the rising relative prices on the utility. As a result, relative welfare falls.

![Graphs showing the effect of increasing $T$ on $\gamma$, $d$, $\omega$ and relative utility and price index, restricted trade scenario](image)

Figure 4: The effect of increasing $T$ on $\gamma$, $d$, $\omega$ and relative utility and price index, restricted trade scenario

To draw conclusions about the effect of IPR protection on individual welfare of the regions, total expenditure conditions for the North and the South should be derived as the sum of labor income and returns on assets in each region. However, for a proper analysis, the scale effect should be removed. This becomes particularly important in the analysis of relative variables in the light of globalization and growing size of the trading South.
4 Conclusion

This paper analyzed a model of North-South trade with the endogenous R&D processes in both regions. North conducts innovating R&D which results in the creation of new varieties, each of a higher quality than the preceding one. South is involved only in copying Northern products, but at a certain lag. The question of interest is the impact of trade and IPR protection on the intensive margin of innovation (size of quality jumps) and the extensive margin of imitation (distance in quality, i.e. quality lag of the South) in a steady-state equilibrium. By describing the mechanism which determines these endogenous variables of technological progress, we attempt to derive the resulting growth and welfare effects of trade and IPR policy and to provide possible answers to some questions brought by globalization. To do so, a model of vertical technological progress is constructed and three alternative scenarios are considered: autarky case as a benchmark, free trade case with information protection ("intensive" IPR policy tool), and finally, restricted trade case where we introduce patents of finite length (extensive form of international IPR protection). The motivation behind the theoretical framework derives from the recent empirical evidence on the North-South trade patterns which reveals the regions’ specialization in different qualities of the same product within industries, and much less in different products or different industries.

Based on analytical and numerical analysis, we find that opening to trade increases the growth rate of both regions while it also results in larger lag in the quality level in the South, implying the appearance of specialization in the South. Furthermore, when comparing the effects of different IPR protection tools in the trading world, we find differences in the magnitude of the changes they bring about in the variables of interest. Also, varying degree of information protection or patent length yield different direction of the change in the relative purchasing power which may even reverse the change in relative welfare when patent length is used as the IPR protection tool. However, common for both policies is that stronger protection decreases world growth rate and increases the quality lag of the South behind the North.

Papers that are interested in analyzing similar questions attempt to address
the issues of globalization through the increase in the size of the South, as more
developing countries join the world trading system. That would be of interest here,
as well. However, the model in this form exhibits scale effects which contradict
empirical growth evidence and thus their removal is crucial for any analysis that
would involve the absolute or relative size of the two regions. In addition, welfare
issues should also be addressed in absolute terms (not only as relative North-South
utility). Furthermore, it might be interesting to derive the optimal patent policy
set by the North and see how it relates to the optimal policy in terms of the world
welfare. Finally, the issues of the transition process towards the steady state in
both regions definitely deserve further studying.
References


