Natural Selection and the Origin of Economic Growth

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August 15, 2000

Abstract

This research develops an evolutionary growth theory that captures the interplay between the evolution of mankind and economic growth since the emergence of the human species. This unified theory encompasses the observed evolution of population, technology and income per capita in the long transition from an epoch of Malthusian stagnation to sustained economic growth. The theory suggests that prolonged economic stagnation prior to the transition to sustained growth stimulated natural selection that shaped the evolution of the human species, whereas the evolution of the human species was the origin of the take-off from an epoch of stagnation to sustained growth.

Keywords: Growth, Technological Progress, Fertility, Human Capital, Evolution, Natural Selection, Malthusian Stagnation.

JEL classification Numbers: O11, O14, O33, O40, J11, J13.

*Galor: Brown University, Hebrew University and CEPR. Moav: Hebrew University. The authors have benefited from discussions with Partha Dasgupta, Eric Gould, John Hassler, Jim Mirrlees, Ariel Rubinstein, Nathan Sussman, and conference participants in the EEA meeting, Bolzano, August 2000, XIth EEA Summer School, Barcelona September 2000, and the Demography and Economic Growth Conference, Harvard University, September 2000.

"*It is not the strongest of the species that survive, nor the most intelligent, but the one most responsive to change.*" Charles Darwin

1 Introduction

This research develops a unified evolutionary growth theory that captures the interplay between the evolution of mankind and economic growth since the emergence of the human species. The theory suggests that prolonged economic stagnation prior to the transition to sustained growth stimulated natural selection that shaped the evolution of the human species, whereas the evolution of the human species had been the origin of the take-off from an epoch of stagnation to sustained growth.

This unified theory encompasses the observed intricate evolution of population, technology and output in the long transition from a Malthusian epoch to sustained economic growth. Consistently with existing evidence, the theory suggests that during the Malthusian era technology evolved rather slowly and population growth prevented a sustained rise in income per capita. Human beings, like other species, have confronted the basic trade-off between offspring's quality and quantity in their implicit Darwinian survival strategies.[1] Although quantity-biased preferences had a positive direct effect on fertility rates, it adversely affected the quality of offspring, their fitness, and hence their fertility rates. The inherent evolutionary pressure in the Malthusian era generated an evolutionary advantage to quality-biased preferences.[2] Natural selection therefore increased the quality of the population inducing faster technological progress that brought about the take-off from the era of stagnation and thereafter a demographic transition that paved the way to sustained economic growth.

The reciprocal relationship between population growth and income per-capita during the era of economic stagnation was examined by Thomas R. Malthus (1798). The Malthusian theory has two essential elements. The first is the existence of some factor of production, such as land, which is in fixed supply, implying decreasing returns to scale for all other factors. The second is a positive effect of the standard of living on the growth rate of population. According

[1] In other species this trade-off is implicit in their biological mechanism.

[2] In this era "the perpetual struggle for room and food" [Thomas R. Malthus (1798, chap. iii. p. 48)] left limited resources for child rearing.

to Malthus, if the standard of living is above the subsistence level, population grows as a natural result of passion between the sexes, whereas if the standard of living is lower than subsistence population declines by either the "preventive check" (i.e., intentional reduction of fertility) or by the "positive check" (i.e., malnutrition, disease, and famine). The Malthusian model implies that, in the absence of changes in the technology or in the availability of land, the size of the population is self-equilibrating. More significantly, even if available resources do expand, the level of income per capita remains unchanged in the long-run; better technology or more land leads to larger, but not richer, population.

The Malthusian description is consistent with the evolution of population and output per capita for most of human history. For thousands of years, the growth rate of output per capita had been negligible over time and the standard of living had not differed greatly across countries.[3] For instance, the average growth rate of GDP per capita in Europe between 500 and 1500 was nearly zero (Angus Maddison ,1982) and real wages in China were lower at the end of the 18th century than they had been at the beginning of the first century (Kang Chao, 1986). Similarly, the pattern of population growth over this era is consistent with the predictions of the Malthusian model. The average annual rate of population growth in Europe between 500 and 1500 was 0.1 percent, and world population grew at an average pace of less than 0.1 percent per year from the year 1 to 1750 (Massimo Livi-Bacci, 1997), reflecting the slow pace of resource expansion and technological progress. Fluctuations in population and wages also bear out the predictions of the Malthusian model. For instance, negative shocks to population, such as the Black Death, were reflected in higher real wages and faster population growth (Livi-Bacci, 1997).[4] Finally, the prediction of the Malthusian theory that differences in technology should be reflected in population density but not in standards of living is also borne out. Prior to 1800 differences in standard of living between countries were quite small by today's standards. And yet there existed wide differences in technology (Richard Easterlin, 1981, Lucas, 1999, and Pritchett, 1997).[5]

[3] As argued by Joel Mokyr (1990), Robert E. Lucas Jr. (1999), and Lant Pritchett (1997), the phenomenon of sustained growth in living standards is only a few centuries old even in the richest countries.

[4] Lee (1997) reports positive income elasticities of fertility and negative income elasticities of mortality from studies examining a wide range of pre-industrial countries. Similarly, Edward A. Wrigley and Roger S. Schofield (1981) find a strong positive correlation between real wages and marriage rates in England over the period 1551-1801.

[5] China's sophisticated agricultural technologies, for example, allowed high per-acre yields, but failed to raise

The emergence from Malthusian stagnation was initially very slow. As observed by Maddison (1982, 1995), the average growth rate of income per capita in Europe was only 0.1 percent per year between 1500 and 1700, and 0.2 percent between 1700 and 1820.[6] As income per capita grew, population growth increased as well to a rate of 0.2 percent in the former period and 0.4 percent in the later period. During this slow transition, the Malthusian mechanism linking higher income to higher population growth continued to function, but the effect of higher population on diluting resources per capita, and thus lowering income per capita, was counteracted by technological progress, which allowed income to keep rising. The take-off from Malthusian stagnation intensified in Europe during the Industrial Revolution and the average growth of output per capita over the period 1820-1870 rose to an annual rate of 1.0 percent.

Fertility rates increased in most of Western Europe until the second half of the nineteenth century, peaking in England and Wales in 1871 and in Germany in 1875. (Tim Dyson and Mike Murphy, 1985, and Ansley J. Coale and Roy Treadway, 1986).[7] Furthermore, the level of resources invested in each child increased as well.[8] Ultimately, the rise in income triggered a demographic transition.[9] Population growth fell and brought about sustained increase in income per capita of 2.2 percent over the period 1929-1990.

This historical evidence suggests that the key events that separate the epoch of Malthusian stagnation and the Sustained Growth Regime are the acceleration in the pace of technological progress and the demographic transition. The emergence from the Malthusian trap and the onset of the demographic transition raise intriguing questions. Why has the link between income per capita and population growth so dramatically reversed? How does one account for

the standard of living above subsistence. Similarly, the introduction of the potato in Ireland in the middle of the 17th century had generated a large increase in population over the century without an improvement in the standard of living. The destruction of this new productive technology by insects has generated a massive decline in population due to the Great Famine and mass migration (Livi-Bacci, 1997).

[6] In the United Kingdom, output per capita grew at an annual rate of 0.4 percent in the 120 years after 1700, while population grew at an annual rate of 0.7 percent.

[7] In addition, as living standards rose, mortality fell. Between the 1740s and the 1840s, life expectancy at birth rose from 33 to 40 in England and from 25 to 40 in France (Livi-Bacci, 1997). Mortality reductions led to growth of the population both because more children reached breeding age and because each person lived for a larger number of years.

[8] For example, the average number of years of schooling in England and Wales rose from 2.3 for the cohort born between 1801 and 1805 to 5.2 for the cohort born 1852-56 and 9.1 for the cohort born 1897-1906. (Robert C. O. Matthews, Charles H. Feinstein, and John C. Odling-Smee, 1982).

[9] The reduction in fertility was most rapid in Europe around the turn of the century. In England, for example, live births per 1000 women aged 15-44 fell from 153.6 in 1871-80 to 109.0 in 1901-10 (Wrigley, 1969). The exception was France, where fertility started to decline in the early 19th century.

the sudden spurt in growth rates? Why had waves of rapid technological progress not generated sustained economic growth in the Pre-Industrial Revolution era? And is there a unified framework of analysis the can account for this intricate evolution of economic growth and mankind since the origin of the human species?

The inconsistency of exogenous as well as endogenous neoclassical growth models with the evolution of economies throughout most of human history has lead recently to the development of unified growth models that are consistent with an epoch of Malthusian stagnation and the transition from Malthusian stagnation to sustained growth. In light of the central role that population growth has apparently played in the Malthusian world as well as in the take-off to sustained growth, these unified models are based on endogenous population growth.[10] In addition they incorporate the main Malthusian features.[11] Oded Galor and David N. Weil (1999, 2000) argue that the inherent positive interaction between population and technology during the Malthusian regime had gradually increased the rate of technological progress, inducing investment in human capital that lead to further technological progress, a demographic transition, and sustained growth.[12] Gary Hansen and Edward Prescott (2000) develop a model

[10]The existing literature on the relation between population growth and output has tended to focus on only one of the regimes described above. The majority of the literature has been oriented toward the modern regime, trying to explain the negative relation between income and population growth either cross-sectionally or within a single country over time (e.g. Robert J. Barro and Gary S. Becker, 1989). Among the mechanisms highlighted in this literature are: (a) higher returns to child quality in developed economies induce a substitution of quality for quantity (Becker, Kevin M. Murphy, and Robert F. Tamura, 1990); (b) developed economies pay higher relative wages for women, thus raising the opportunity cost of children (Oded Galor and David N. Weil, 1996); (c) the net flow of transfers from parents to children grows (and possibly switches from negative to positive) as countries develop (John W. Caldwell, 1976); (d) higher fertility rates among unskilled workers increase the return to skills and an incentive to substitute quality for quantity (Momi Dahan and Daniel Tsiddon, 1998). Recent papers that are concerned with the Malthusian regime are Kremer (1993) and Lucas (1999). Kremer (1993) models a reduced form interaction between population and technology along a Malthusian equilibrium, and Lucas presents a Malthusian model in which households optimize over fertility and consumption.

[11]Models that are not based on Malthusian elements are unable to capture the long epoch of Malthusian stagnation in which the output per capita fluctuates around a subsistence level. For instance, an interesting research by Marvin Goodfriend and John McDermott (1995) demonstrate that exogenous population growth increases population density and hence generates a greater scope for the division of labor inducing the development of markets and economic growth. Their model, however, lacks Malthusian elements and counterfactually it implies therefore that since the emergence of a market economy over 5000 years ago growth has been strictly positive.

[12]Michael Kremer (1993) examines the co-evolution of population and technology since one million BC to the present, providing creative evidence for the importance of the scale effect in economic growth. Based upon a reduced form relationship between technology and population he studies the evolution of these two variables during either the Malthusian regime when output per capita is at subsistence, or in an environment when growth in output per capita is positive and monotonically increasing over time. Unlike the described unified models, however, there is no mechanism that elevates the economy from the absorbing Malthusian equilibrium. Namely, sustained growth is feasible only if the economy has been growing throughout human history. Furthermore, to generate a demographic transition additional ad-hoc structure is required.

in which an exogenous technological progress in a latent industrial technology along with an assumed hump-shaped evolution of population growth in the process of development brings about a transition from a stagnating agricultural economy to a growing industrial economy. Charles I. Jones (2000) suggests that the virtuous circle between the size of the population and the production of ideas along with the improvement in institutions that promote innovation have lead to the transition from stagnation to growth.[13]

This research, in contrast, develops a unified evolutionary theory that focuses on the interaction between the evolution of the human species and the transition from a Malthusian Epoch to sustained growth. The fundamental premise that has guided this research is that, due to natural selection, the composition of characteristics of the human species that may be highly relevant for the understanding of the origin of economic growth has not been stationary since the emergence of the human species. The study focuses mostly on the change in the composition of types within the Homo Sapiens (i.e., variants within the species) rather than the more dramatic evolution from the Homo Erectus to the Homo Sapiens, for instance. Namely, the theory focuses on the evolution in the composition of types within a population that has only a modest variety in genetic traits across types. The theory abstracts therefore from the evolution in the size of the human brain, focusing on the evolution of preferences within the Homo Sapiens. [14] Evidence regarding evolutionary process in nature suggests that evolutionary processes in the composition of types is rather rapid.[15]

Unlike previous evolutionary models in which population growth among types of the human species is assumed to be an increasing function of fitness (e.g., income or consumption)

[13]Recent growth models with endogenous fertility of the long transition from stagnation to growth also include Olivier Morand (2000), Nils Lagerlof (2000), Robert Tamura (2000), among others.

[14]The evolution from the Homo Erectus to Homo Sapiens, in contrast, in which brain size nearly doubled, had taken more than 1 million years. In contrast to the clear evolutionary trade-off that is introduced by the choice between quality and quantity of offspring, a focus on the evolution in brain size appears somewhat less interesting from an economic viewpoint. In particular, from the Neolithic period and till the demographic transition it appears that higher intelligence had no obvious evolutionary trade-off; Higher intelligence had been associate with higher potential income and had generated an absolute evolutionary advantage. In a sequel to this paper, Galor and Moav (2000) develop a unified theory that focuses on the evolution of intelligence and the origin of economic growth. As is established in this study, a quality-quantity trade-off is a necessary condition for the demographic transition.

[15]For instance, H. B. D. Kettlewell (1973) 's field experiments on industrial melanism in the peppered moth, Biston Betularia, has shown that given its typical background of a white tree trunk this moth has been white with a small population of black mutants. However, in areas in which industrial black carbon changed the background color the entire population turned black within a short time period.

and is thus indistinguishable from population growth among other species, in the proposed theory fertility decisions which are based on the optimization of the household generate a non-monotonic relationship between population growth and income.[16] This fundamental distinction enables the theory to capture the monotonic evolution of the population growth and income per capita until the 19th century as well as the reversal in this relationship during the demographic transition paving the way to sustained economic growth. Furthermore, the integration between an evolutionary process and a unified growth model generates an endogenous take-off from an epoch of Malthusian stagnation based on the evolution of mankind.

The theory is based on four fundamental elements. The first element of the model consists of the main ingredients of a Malthusian world. The economy is characterized by a fixed factor of production, land, and a subsistence consumption constraint below which individuals cannot survive. If technological progress permits output per worker to exceed the subsistence level of consumption, population rises, the land-labor ratio falls, and in the absence of further technological progress wages fall back to the subsistence level. Income per capita is therefore self-equilibrating and the economy is in a Malthusian stagnation. Sustained technological progress, however, can overcome the offsetting effect of population growth, by increasing effective resources per capita (i.e. the combined input of technology and land per capita), allowing sustained income growth.

In the Malthusian era, therefore, human beings struggled for survival and their fertility rates had been positively influenced by their excess income over the subsistence level of consumption. Differences in income produced therefore differences in fertility rates across individuals. Moreover, if differences in income across individuals reflected differences in genetic

[16] The Darwinian methodology has been employed in a sequence of insightful studies about the evolution of preferences (e.g., Ingemar Hansson and Charles Stuart, 1990, Alan Rogers, 1994, and Theodore Bergstrom, 1995 among others.) The focus of these models is fundamentally different. They are primarily designed to explain the determination of preference. The closest evolutionary model to the context of economic development is developed by Ingemar Hansson and Charles Stuart (1990). They demonstrate that in a Malthusian environment, from which the economy never escapes, evolution selects individuals with time preference (and hence saving) which is closest to the golden rule. Although the Malthusian setting has no important role in the determination of the type with an evolutionary advantage, it enables the authors to fix the size of the population (for a given level of technology) and hence to eliminate types with evolutionary disadvantage. In contrast, in our unified theory the Malthusian pressure is the prime determinant of the type with the evolutionary advantage. Furthermore, the evolution of the composition of types in our model brings about the take-off from the Malthusian regime, demographic transition and sustained growth, which are absent in all other evolutionary models.

traits (e.g., preferences, and physical or intellectual ability), then the effect of the Malthusian pressure on fertility rates would affected the genetic composition of the population.

The second and most novel element of the model incorporates the main ingredients of the Darwinian world (i.e., variety, natural selection, and evolution) in a Malthusian economic environment. It demonstrates the importance of the Malthusian pressure for the evolution of the human species. The economy is populated with individuals whose preferences reflect the implicit Darwinian survival strategy. Although individuals do not operate consciously so as to assure the evolutionary advantage of their type (i.e., their variant within the species), the existence of variety of types enables nature to select those who fit the economic environment, increasing the likelihood of the survival of the human species in a changing world.

Inspired by fundamental components of the Darwinian theory, individuals' preferences are defined over consumption above a subsistence level as well as over the quality and the quantity of their children. These simple and commonly employed preferences may be viewed as the manifestation of the Darwinian survival strategy and represents the most fundamental trade-off that exists in nature. Namely, the trade-off between resources allocated to the parent and the offspring, and the trade-off between the number of offspring and the resources allocated to each offspring.

The subsistence consumption constraint assures the mere physiological survival of the parent and hence increases the likelihood of the survival of the lineage (dynasty). Resources allocated to parental consumption beyond the subsistence level raise the parental labor productivity and resistance to adverse shocks (e.g., famine, disease, and variability in output), generating a positive effect on the fitness of the parent and the survival of the lineage. This positive effect, however, is counterbalanced by the implied reduction in the resources allocated to the offspring, generating a negative affect on the survival of the lineage.

The significance that the individual attributes to child quantity as well as child quality reflects the well known variety in the quality-quantity survival strategies that exists in nature. Human beings, like other species, confront the basic trade-off between offspring's quality and quantity in their implicit Darwinian survival strategies. Although a quantity-biased preference has a positive effect on fertility rates and may therefore generate an evolutionary advantage, it adversely affects the quality of offspring, their fitness, and their income. Hence, in the pre-demographic transition era, when fertility rates are positively associated with income levels, it may generate an evolutionary disadvantage.

The economy consists of a variety of types of individuals distinguished by the weight given to child quality in their preference. The household chooses the number of children and their quality in the face of a constraint on the total amount of resources that can be devoted to child-raising and labor market activities.[17] Preferences are hereditary and hence the distribution of types evolves over time due to the effect of natural selection.[18] The economic environment determines the type with the evolutionary advantage (i.e., the type characterized by higher fertility rates). In the pre-demographic transition era, when fertility rates are positively associated with income levels, the Malthusian pressure generates an evolutionary advantage to individuals whose preferences are biased towards child quality increasing their representation in the population. In the post-demographic transition era, however, due to the endogenous evolution in the economic environment, individuals whose preferences are biased towards child quantity has the evolutionary advantage.

The third element of the model links the evolution of the human species to the process of economic growth. Following the well-documented and commonly employed hypothesis, human capital is assumed to have a positive effect on technological progress and therefore on economic growth.[19] Hence, natural selection and the implied evolution in the composition of types that brings about an increase in the representation of individuals whose preferences are biased towards child quality, has a positive effect on the average quality of the population and therefore on the rate of technological progress. In particular, the Malthusian pressure that increases the representation of individuals whose preferences are biased towards child quality in the population generates acceleration in technological progress.

The fourth element links the rise in the rate of technological progress to the demographic

[17]This standard approach to household fertility behavior is based on Becker (1981).

[18]For simplicity, the model abstracts from marriages. Namely, each offspring has a single parent.

[19]This link between education and technological change was proposed by Richard R. Nelson and Edmund S. Phelps [1966]. For supportive evidence see Easterlin (1981) and Mark Doms, Timothy Dunne, and Kenneth R. Troske (1997). In order to focus on the role of the evolutionary process in the demographic transition and modern growth, the model abstracts from the potential positive effect of the overall size of the population on the rate of technological progress. As discussed in the concluding remarks, adding this scale effect would simply accelerate the transition process. Evidence regarding the role of the scale of the economy in technological progress is mixed. While Kremer (1993) provides some supporting historical evidence, Jones (1995) argues that in the 20th century it appears that there is no evidence for a scale effect.

transition and sustained economic growth. A rise in the rate of technological progress is assumed to increase the rate of return to human capital, inducing parents to substitute child quality for child quantity.[20] The argument that technological progress itself raises the return to human capital was most clearly stated by Richard Nelson and Edmund Phelps (1966) and Theodore W. Schultz (1964).[21] Although the new technological level may reflect in the long-run either a skill-biased or skill-saving technological change, it is argued that the transition to the new technological state is mostly skill biased in the short-run.[22] Technological progress reduces the adaptability of existing human capital for the new technological environment. Education, however, lessens the adverse effects of technological progress. That is, skilled individuals have a comparative advantage in adapting to the new technological environment.

Technological progress has therefore two effects on the evolution of population. First, it increases the return to human capital, inducing parents to raise the quality of each child and reduce the number of children. But, second, by raising parental income above the subsistence level, technological progress provides more resources for quality as well as quantity of children. Hence, an increase in the rate of technological progress increases the average quality in the population, further accelerating technological progress. Ultimately, technological progress becomes sufficiently rapid so as to induce a reduction in fertility rates, generating a demographic transition and sustained economic growth.

The interaction between these four fundamental elements generates an evolutionary pattern that is consistent with the observed evolution of the world economy and the human population from Malthusian stagnation to sustained growth.

Suppose that in the early era in the history of mankind, the population of the world consisted of homogeneous individuals of the "quantity type" who place low weight on the quality of their offspring. Given the initial conditions, the economy is in a locally stable

[20]Unlike Gray Becker (1981) in which a high level of income is inducing parents to switch to having fewer, higher quality children, the substitution of quality for quantity in this paper is in response to technological progress.

[21]Schultz (1975) cites a wide range of evidence in support of this theory. Similarly, Andrew Foster and Mark Rosenzweig (1996) find that technological change during the green revolution in India raised the return to schooling, and that school enrollment rates responded positively to this higher return.

[22]If technological changes are skill-biased in the long run, then the effect will be enhanced, while if technology is skill-saving then our effect will be diluted. Goldin and Katz (1998) provide evidence regarding technology-skill complementarity that is consistent with our short-run view of skill-biased technological change as well as the long-run view.

Malthusian steady-state equilibrium where technology is stationary, parents have no incentive to raise quality children, and hence the level of human capital, effective resources, output per capita, and population are constant as well. Deviations from this steady-sate equilibrium, due to some exogenous shocks to population or resources are undone in a classic Malthusian fashion. They induce temporary changes in the real wage and fertility, which in turn drive income per capita back to its stationary equilibrium level.

Mutation introduces a very small number of individuals of "the quality type"- who place higher weight on the quality of their children.[23] Subsequently, in every period the economy consists of two types of individuals: individuals of the "quality type" - with a higher weight for quality, and individuals of the "quantity type" - with a lower weight for quality. In the initial periods after the mutation affects the economy the fraction of individuals of the quality type is small, the rate of technological progress is slow, inducing little investment in quality, and resulting in proportional increases in output and population. The economy is in the vicinity of a temporary locally stable Malthusian steady-state equilibrium.

In the early Malthusian era, when humans merely struggle for survival, individuals with a preference bias towards quality of offspring have an evolutionary advantage over individuals of the quantity type. That is, the fraction of individuals of quality type rises in the population, despite their preference bias against the quantity of their offspring. Hence, in early stages of development the Malthusian pressure provides an evolutionary advantage to the quality type. The income of individuals of the quantity type is near subsistence and fertility rates are therefore near replacement level. In contrast, the wealthier, quality type, can afford higher fertility rates (of higher quality offspring). The fraction of individuals of the quality type in the population increases monotonically over this Malthusian regime, generating higher rates of technological progress.

[23]One should not be concerned about the possibility that this mutation would have an evolutionary advantage much earlier in history. This is a simplifying assumption that is designed to capture a sequence of mutations which result in a gradual increase in the variance in the distribution of the quality parameter. This process has for a long period no effect on the quality composition of the population, since in the absence of technological progress there is a large range of the quality parameter for which individuals choose no investment in child quality. Ultimately mutations increase the variance sufficiently and individuals of type a - who invest in quality even in the absence of technological change - emerge. Clearly, the existence of heterogeneity of types throughout human history would not affect the qualitative analysis as long as the fraction of the quality type is initially small. The focus on two types of individuals simplifies the exposition considerably and permits an analytical solution of the evolution of this complex three-dimensional system.

As the fraction of individuals of the quality type increases, technological progress intensifies, and ultimately the dynamical system changes qualitatively, the Malthusian temporary steady-state vanishes endogenously and the economy takes-off from the Malthusian trap. The positive feedback between the rate of technological progress and the level of education reinforces the growth process, setting the stage for the Industrial Revolution. The increase in the rate of technological progress brings about two effects on the evolution of population and its quality. On the one hand, improved technology eases households' budget constraints, providing more resources for quality as well as quantity of children. On the other hand, it induces a reallocation of these increased resources toward child quality. Hence, an increase in the rate of technological progress increases the average quality in the population, further accelerating technological progress. In the early stages of the transition from the Malthusian regime the effect of technological progress on the parental budget constraint dominates, and the population growth rate as well as the average quality increases. Ultimately, however, technological progress becomes sufficiently rapid so as to induce a reduction in fertility rates, generating a demographic transition in which the rate of population growth declines along with an increase in the average level of education. The economy converges to a steady-state equilibrium with sustained growth of output per worker.

During the transition from the Malthusian stagnation to the sustained growth regime, once the economic environment improves sufficiently the evolutionary pressure weakens, the significance of quality for survival (fertility) declines, and individuals of the quantity type gain the evolutionary advantage. Namely, as technological progress brings about an increase in income, the Malthusian pressure relaxes, and the domination of wealth in fertility decisions diminishes. The inherent advantage of the quantity type in reproduction gradually dominates and fertility rates of the quantity type ultimately overtake those of the quality type. The fraction of individuals of the quality type starts declining and the long run equilibrium is dominated by the quantity type. Nevertheless, the growth rate of output per worker may remain positive, although at a lower level than the one existed in the peak of the transition.

2 The Basic Structure of the Model

Consider an overlapping generation economy in which economic activity extends over infinite discrete time. In every period the economy produces a single homogenous good using land and efficiency units of labor as inputs. The supply of land is exogenous and fixed over time, whereas The supply of efficiency units of labor is determined by households' decisions in the preceding period regarding the number and the level of human capital of their children.

2.1 Production of Final Output

Production occurs according to a constant-returns-to-scale technology that is subject to endogenous technological progress. The output produced at time t, Y_t, is

$$Y_t = H_t^{1-\alpha}(A_t X)^{\alpha} \tag{1}$$

where H_t is the aggregate quantity of efficiency units of labor at time t, X is land employed in production, $A_t > 0$, represents the endogenously determined technological level at time t, and $\alpha \in (0,1)$. The multiplicative form in which the level of technology, A_t, and land, X, appear in the production function implies that the relevant factor for the output produced is the product of the two, defined as "effective resources."

Suppose that there are no property rights over land. The return to land is therefore zero, and the wage per efficiency unit of labor, w_t, is therefore equal to the output per efficiency unit of labor produced at time t. Hence,

$$w_t = x_t^{\alpha} \tag{2}$$

where $x_t \equiv A_t X / H_t$ denotes effective resources per efficiency unit of labor at time t.

The modeling of the production side is based on two simplifying assumptions. First, capital is not an input in the production function, and second the return to land is zero.[24]

[24] Alternatively one could have assumed that the economy is small and open to a world capital market in which the interest rate is constant. In this case, the quantity of capital will be set to equalize its marginal product to the interest rate, while the price of land will follow a path such that the total return on land (rent plus net price appreciation) is also equal to the interest rate. This is the case presented in Galor and Weil (1998). As discussed previously, capital has no role in the mechanism that is underlined in this paper, and the qualitative results would not be affected if the supply of capital were endogenously determined. Allowing for capital accumulation and property rights over land would complicate the model to the point of intractability.

2.2 Individuals

In each period a new generation of individuals is born. Each individual has a single parent. Members of generation t (those who join the labor force in period t) live for two periods. In the first period of life (childhood), $t-1$, individuals consume a fraction of their parent's time. The required time increases with children's quality. In the second period of life (parenthood), t, individuals are endowed with one unit of time, which they allocate between child rearing and labor force participation. They choose the optimal mixture of quantity and quality of children and supply their remaining time in the labor market, consuming their wages.

2.2.1 Preferences and Budget Constraints

Every generation t consists of a variety of individuals (type i of generation t) distinguished by the trade-off between child quality and quantity in their preference. Individuals *within* a dynasty are of the same type. That is, preferences are hereditary and they are transmitted without alteration from generation to generation within a dynasty. In the absence of changes in the economic environment, individuals within a dynasty would remain identical over time, whereas individuals across dynasties might differ in their types and therefore in their quality (education) and income.

The distribution of types evolves over time due to the effect of natural selection on the relative size of each dynasty. The type with the evolutionary advantage (i.e., the type characterized by higher fertility rates) is determined by the economic environment, and it may be replaced due to the endogenous evolution in this environment. Although a quantity-biased preference has a positive effect on fertility rates and may therefore generate an evolutionary advantage, it adversely affects the quality of offspring and their potential income, and in the pre-demographic transition era, when fertility rates are positively associated with income levels, it may generate an evolutionary disadvantage.

Individuals' preferences reflect the implicit Darwinian survival strategy. Although individuals do not operate consciously so as to assure the evolutionary advantage of their type, the variety of types (that is the outcome of mutations in the initial stage) assures, via natural selection, the survival of the human spices.

The preferences of members of generation t are defined over consumption above a subsistence level as well as over the quality and the quantity of their children. These simple and commonly employed preferences may be viewed as the manifestation of the Darwinian survival strategy. The subsistence consumption constraint assures the mere physiological survival of the family and hence the survival of the lineage (dynasty). Moreover, consumption beyond the subsistence level positively affects the fitness of individuals due to the rise in their resistance to adverse shocks (e.g., famine, disease, and variability in output), and the beneficial effect of improved nourishments on labor productivity.[25] The significance that the individual attributes to child quantity as well as child quality reflects the well known variety in the quality-quantity survival strategies that exists in nature. Although a quantity-biased preference has a positive effect on fertility rates and may therefore generate an evolutionary advantage, it adversely affects the quality of offspring and the potential fitness of each child.

The preferences are represented by the utility function defined over consumption above a subsistence level $\tilde{c} > 0$, as well as over the quality of their children (measured by their potential income) and the quantity of their children.

$$u_t^i = (1-\gamma)\ln c_t^i + \gamma[\ln n_t^i + \beta^i \ln w_{t+1} h_{t+1}^i]; \qquad \gamma \in (0,1) \tag{3}$$

where c_t^i is the household consumption of a type i individual of generation t, n_t^i is the number of children, h_{t+1}^i is the level of human capital of each child, w_{t+1} is the wage per efficiency unit of labor at time $t+1$ and $\beta^i \in (0,1]$ is the relative weight given to quality in the preference of dynasty i. The quality-parameter, β^i, is transmitted from generation to generation within a dynasty and remains stationary across time.[26] The utility function is strictly monotonically increasing and strictly quasi-concave, satisfying the conventional boundary conditions that assure, for sufficiently high income, the existence of an interior solution for the utility maximization problem. However, for a sufficiently low level of income the subsistence consumption constraint is binding and there is a corner solution with respect to the consumption level.[27]

[25] For simplicity, it is assumed that the subsistence consumption constraint and the weight given to consumption in the utility function are homogenous across individuals and hence they are not subjected to natural selection and Darwinian evolution.

[26] As will become apparent, the distribution of β^i changes due to the effect of natural selection on the distribution of types within each generation. Furthermore, although β^i is stationary across time within a dynasty, the optimization of individuals within a dynasty changes across time due to changes in the economic environment.

[27] As will become apparent, the presence of a subsistence consumption constraint provides the Malthusian piece of our model. The formulation that we use implicitly stresses a "demand" explanation for the positive

Following the standard model of household fertility behavior (Becker, 1981), the household chooses the number of children and their quality in the face of a constraint on the total amount of time that can be devoted to child-raising and labor market activities. We further assume that the only input required to produce both child quantity and child quality is time.[28]

Let $\tau^n + \tau^e e^i_{t+1}$ be the time cost for a member i of generation t of raising a child with a level of education (quality) e^i_{t+1}. That is, τ^n is the fraction of the individual's unit time endowment that is required in order to raise a child, regardless of quality, and τ^e is the fraction of the individual's unit time endowment that is required for each unit of education for each child. The time required in order to raise a child, regardless of quality is assumed to be sufficiently small so as to assure that population can have a positive growth rate. That is, $\tau^n < \gamma$.

Consider a member i of generation t who is endowed with h^i_t efficiency units of labor at time t. Define potential income, z^i_t, as the potential earning if the entire time endowment is devoted to labor force participation:

$$z^i_t \equiv w_t h^i_t = x^\alpha_t h^i_t \equiv z(x_t, h^i_t) \tag{4}$$

Potential income is divided between expenditure on child rearing (quantity as well as quality), at an opportunity cost of $w_t h^i_t[\tau^n + \tau^e e^i_{t+1}]$ per child, and consumption, c^i_t. Hence, in the second period of life (parenthood), the individual faces the budget constraint:

$$w_t h^i_t n^i_t(\tau^n + \tau^e e^i_{t+1}) + c^i_t \le w_t h^i_t \equiv z^i_t. \tag{5}$$

2.2.2 The Production of Human Capital

Individuals' level of human capital is determined by their quality (education) as well as by the technological environment. Incorporating the insight of Nelson and Phelps (1966) and of Schultz (1964) previously discussed, technological progress is assumed to raise the value of education in producing human capital. In particular, the time required for learning the new

income elasticity of population growth at low income levels, since higher income will allow individuals to afford more children. However, one could also cite "supply" factors, such as declining infant mortality and increased natural fertility, to explain the same phenomenon. See Nancy Birdsall (1988) and Randall J. Olsen (1994).

[28]If both time and capital are required in order to produce child quality and if the capital cost is fully indexed to the wage in the economy, the analysis remains intact. Otherwise, as will become apparent, the qualitative analysis remains intact. In particular, if the capital cost rises less than wages, the transition would be intensified. As the economy develops and wages increase, the relative cost of a quality child will diminish and individuals will substitute quality for quantity of children.

technology diminishes with the level of education and increases with the rate of technological change. Hence, technological progress increases the return to education.

The level of human capital of children of a member i of generation t, h^i_{t+1}, is an increasing function of their education, e^i_{t+1}, and a decreasing function of the rate of progress in the state of technology from period t to period $t+1$, $g_{t+1} \equiv (A_{t+1} - A_t)/A_t$. The higher is children's quality, e^i_{t+1}, the smaller is the adverse effect of technological progress.

$$h^i_{t+1} = h(e^i_{t+1}, g_{t+1}). \tag{6}$$

where $h(e^i_{t+1}, g_{t+1}) > 0, h_e(e^i_{t+1}, g_{t+1}) > 0,\ h_{ee}(e^i_{t+1}, g_{t+1}) < 0,\ h_g(e^i_{t+1}, g_{t+1}) < 0, h_{gg}(e^i_{t+1}, g_{t+1}) > 0,\ h_{eg}(e^i_{t+1}, g_{t+1}) > 0,\ \forall (e^i_{t+1}, g_{t+1}) \geq 0$, $\lim_{g\to\infty} h(0, g_{t+1}) = 0$ and $h(0,0) = 1$.

Hence, the individual's level of human capital is an increasing, strictly concave function of education, and a decreasing strictly convex function of the rate of technological progress. Furthermore, education lessens the adverse effect of technological progress. That is, technology complements skills in the production of human capital.

Although the potential number of efficiency units of labor is diminished due to the transition from the existing technological state to a superior one - the 'erosion effect', each individual operates with a superior level of technology - the 'productivity effect'.[29] Moreover, once the *rate* of technological progress reaches a positive steady-state level, the 'erosion effect' is constant, whereas productivity grows at a constant rate.

2.2.3 Optimization

Members of generation t choose the number and quality of their children, and therefore their own consumption, so as to maximize their intertemporal utility function. Substituting (5)-(6) into (3), the optimization problem of a member i of generation t is:

$$\{n^i_t, e^i_{t+1}\} = argmax\{(1-\gamma)\ln w_t h^i_t[1 - n^i_t(\tau^n + \tau^e e^i_{t+1})] + \gamma[\ln n^i_t + \beta^i \ln w_{t+1} h(e^i_{t+1}, g_{t+1})]\} \tag{7}$$

subject to

$$w_t h^i_t[1 - n^i_t(\tau^n + \tau^e e^i_{t+1})] \geq \tilde{c};$$

$$(n^i_t, e^i_{t+1}) \geq 0.$$

[29] See Galor and Moav (2000).

The optimization with respect to n_t^i implies that as long as potential income of a member i of generation t is sufficiently high so as to assure that $c_t^i > \tilde{c}$, the time spent by individual i raising children is γ, while $1-\gamma$ is devoted for labor force participation. However, for low levels of potential income, the subsistence constraint binds. The individual devotes a sufficient fraction of the time endowment for labor force participation so as to assure consumption of the subsistence level, $\tilde{c}$, and uses the rest of the time endowment for child rearing.

Let $\tilde{z}$ be the level of potential income at which the subsistence constraint is just binding. That is, $\tilde{z} \equiv \tilde{c}/(1-\gamma)$. It follows that for $z_t^i \geq \tilde{c}$

$$n_t^i[\tau^n + \tau^e e_{t+1}^i] = \begin{cases} \gamma & if \quad z_t^i \geq \tilde{z}; \\ 1 - [\tilde{c}/z_t^i] & if \quad z_t^i < \tilde{z}. \end{cases} \tag{8}$$

If $z_t^i < \tilde{c}$, then $n_t^i = 0$ and type i becomes extinct.

It should be noted that for a given level of potential income, $z_t^i = x_t^\alpha h_t^i$, the parameter β^i, does not affect the time allocation between child rearing and labor force participation. It affects, however, the division between time spend on child quality and time devoted to child quantity. As will become apparent, individuals with a higher β^i spend more time on child quality on the account of lower quantity.

As long as the potential income of a member i of generation t, z_t^i, is below $\tilde{z}$, then the fraction of time necessary to assure subsistence consumption, $\tilde{c}$, is larger than $1-\gamma$ and the fraction of time devoted for child rearing is therefore below γ. As the wage per efficiency unit of labor increases, the individual can generate the subsistence consumption with smaller labor force participation and the fraction of time devoted to child rearing increases.[30]

Figure 1 shows the effect of an increase in potential income z_t^i on the individual's choice of total time spent on children and consumption. The income expansion path is vertical until the level of income passes the critical level that permits consumption to exceed the subsistence level. Thereafter, the income expansion path becomes horizontal at a level γ in terms of time

[30] John D. Durand (1975) and Goldin (1994) report that, looking across a large sample of countries, the relationship between women's labor force participation and income is U-shaped. The model presented here explains the negative effect of income on labor force participation for poor countries, and further predicts that this effect should no longer be operative once potential income has risen sufficiently high. It does not, however, explain the positive effect of income on participation for richer countries. See, however, Galor and Weil (1996) for a model that does explain this phenomenon.

devoted for child rearing.

Regardless of whether potential income is above or below $\tilde{z}$, increases in wages will not change the division of child-rearing time between quality and quantity. What *does* affect the division between time spent on quality and time spent on quantity is the rate of technological progress, as well as the preference for quality, β^i. Specifically, using (8), the optimization with respect to e^i_{t+1} implies that independently of the subsistence consumption constraint the implicit functional relationship between e^i_{t+1} and g_{t+1} as derived in Lemma 1 is given by

$$G(e^i_{t+1}, g_{t+1}; \beta^i) \equiv \beta^i(\tau^n + \tau^e e^i_{t+1})h_e(e^i_{t+1}, g_{t+1}) - \tau^e h(e^i_{t+1}, g_{t+1}) \begin{cases} = 0 & if \quad e^i_{t+1} > 0 \\ \leq 0 & if \quad e^i_{t+1} = 0 \end{cases} \tag{9}$$

where $G_e(e_{t+1}, g_{t+1}; \beta^i) < 0$, $G_g(e_{t+1}, g_{t+1}; \beta^i) > 0$ and $G_\beta(e_{t+1}, g_{t+1}; \beta^i) > 0$ $\forall g_{t+1} \geq 0$, and $\forall e_{t+1} \geq 0$.

Since $G(0, 0; 0) < 0$, it follows that individuals with a sufficiently low level of β^i do not invest in the human capital of their offspring when the future rate of technological progress is zero. To assure that individuals with a sufficiently high level of β^i would invest in the human capital of their offspring even when the rate of technological progress is 0, it is sufficient to assume that

$$G(0, 0; 1) = \tau^n h_e(0, 0) - \tau^e h(0, 0) > 0. \tag{A1}$$

Let $\underline{\beta}$ denote the threshold level of the quality parameter above which individuals of type i of generation t invests in the education of their offspring even when $g_{t+1} = 0$. That is, $G(0, 0; \underline{\beta}) = 0$. Hence, as follows from the properties of (9), there exists $\underline{g}(\beta^i) \geq 0$ such that $G(0, \underline{g}(\beta^i), \beta^i) = 0$ for all $\beta^i \leq \underline{\beta}$.

Lemma 1 *Under Assumption A1,*
The quality of children, e^i_{t+1}, chosen by a member i of generation t is an increasing function of g_{t+1} and β^i,

$$e^i_{t+1} = \varepsilon(g_{t+1}; \beta^i) \equiv e^i(g_{t+1}) \begin{cases} = 0 & if \quad g_{t+1} \leq \underline{g}(\beta^i) \text{ and } \beta^i \leq \underline{\beta} \\ > 0 & if \quad g_{t+1} > \underline{g}(\beta^i) \text{ or } \beta^i > \underline{\beta} \end{cases}$$

where, $\varepsilon_g(g_{t+1}; \beta^i) > 0$ *and* $\varepsilon_\beta(g_{t+1}; \beta^i) > 0$ $\forall g_{t+1} > \underline{g}(\beta^i)$ *and* $\forall \beta^i > \underline{\beta}$.

Proof.

The Proof follows from the properties of (6), (9) and A1 noting that

$\varepsilon_g(g_{t+1}, \beta^i) = -G_g(e_{t+1}, g_{t+1}, \beta^i)/G_e(e_{t+1}, g_{t+1}, \beta^i) > 0$

and that

$\varepsilon_\beta(g_{t+1}, \beta^i) = -G_\beta(e_{t+1}, g_{t+1}, \beta^i)/G_e(e_{t+1}, g_{t+1}, \beta^i) > 0.$ □

As is apparent from (9), $\varepsilon_{gg}(g_{t+1}; \beta^i)$ depends upon the third derivatives of the production function of human capital. A concave reaction of the level of education to the rate of technological progress appears plausible economically, hence it is assumed that

$$\varepsilon_{gg}(g_{t+1}; \beta^i) < 0 \quad \forall g_{t+1} > \underline{g}(\beta^i) \text{ and } \forall \beta^i > \underline{\beta}. \tag{A2}$$

As follows from Lemma 1, the level of human capital of an individual of type i in period $t+1$ is therefore

$$h^i_{t+1} = h(e^i_{t+1}, g_{t+1}) = h(\varepsilon(g_{t+1}; \beta^i), g_{t+1}) = h(e^i(g_{t+1}), g_{t+1}) \equiv h^i(g_{t+1}). \tag{10}$$

As is apparent from (9) and the properties of (6), $\partial h^i(g_t)/\partial g_t$ can be positive or negative. Since the response of education, e_{t+1}, to g_{t+1} may be viewed as a measure intended to offset the erosion effect of g_{t+1} on the level of human capital, it is natural to assume that $\forall i$[31]

$$\partial h^i(g_t)/\partial g_t < 0 \qquad \forall g_{t+1} > 0. \tag{A3}$$

Furthermore, substituting $e^i_{t+1} = \varepsilon(g_{t+1}; \beta^i)$ into (8), noting that $z^i_t = x^\alpha_t h(\varepsilon(g_t; \beta^i), g_t) = x^\alpha_t h^i(g_t)$, it follows that for $z^i_t \geq \tilde{c}$,

$$n^i_t = \begin{cases} \gamma/[\tau^n + \tau^e \varepsilon(g_{t+1}; \beta^i)] & if \quad z^i_t \geq \tilde{z} \\ (1 - [\tilde{c}/z^i_t])/[\tau^n + \tau^e \varepsilon(g_{t+1}; \beta^i)] & if \quad z^i_t \square \tilde{z}. \end{cases} \equiv n(g_{t+1}, z^i_t; \beta^i) = n(g_{t+1}, z(x_t, h^i(g_t)); \beta^i), \tag{11}$$

where, $\partial n(g_{t+1}, z(x_t, h^i(g_t)); \beta^i)/\partial x_t > 0$ and $\partial^2 n(g_{t+1}, z(x_t, h^i(g_t)); \beta^i)/\partial x^2_t < 0$ $\forall x_t < [\tilde{z}/h^i(g_t)]^{1/\alpha}$, [32] and $\partial n(g_{t+1}, z(x_t, h^i(g_t)); \beta^i)/\partial x_t = 0$ $\forall x_t \geq [\tilde{z}/h^i(g_t)]^{1/\alpha}$.

[31] This assumption narrows the number of scenarios explored, but has no qualitative significance for the central hypothesis.

[32] $[\tilde{z}/h^i(g_t)]^{1/\alpha}$ is the level of x_t, given g_t, such that the consumption constraint is just binding for type i.

The following proposition summarizes the properties of the functions $\varepsilon(g_{t+1};\beta^i)$, and $n(g_{t+1}, z_t^i;\beta^i)$ and their significance for the evolution in the substitution of child quality for child quantity in the process of development:

Proposition 1 *Under A1,*

1. *Technological progress results in a decline in the parents' chosen number of children and an increase in their quality (i.e.,* $\partial n_t^i/\partial g_{t+1} \square 0,$ *and* $\partial e_{t+1}^i/\partial g_{t+1} \geq 0;$*).*

2. *If parental potential income is below* $\tilde{z}$ *(i.e., if the subsistence consumption constraint is binding), an increase in parental potential income raises the number of children, but has no effect on their quality (i.e.,* $\partial n_t^i/\partial z_t^i > 0,$ *and* $\partial e_{t+1}^i/\partial z_t^i = 0 \quad$ *if* $z_t^i < \tilde{z}$*).*

3. *If parental potential income is above* $\tilde{z}$*, an increase in parental potential income does not change the number of children or their quality (i.e.,* $\partial n_t^i/\partial z_t^i = \partial e_{t+1}^i/\partial z_t^i = 0$ *if* $z_t^i > \tilde{z}$*).*

Proof. Follows directly from Lemma 1 and (11). □

It follows from Proposition 1 that if the subsistence consumption constraint is binding, an increase in the effective resources per worker raises the number of children, but has no effect on their quality, whereas if the constraint is not binding, an increase in the effective resources per worker does not change the number of children or their quality. Hence, for a given rate of technological change, parental type, rather than parental income, is the sole determinant of offspring's quality.

2.3 The Distribution of Types and Human Capital

In period 0 there are L_0^a identical adult individuals of type a - "the quality type" - with a high quality-parameter $\beta^a > \underline{\beta}$, and L_0^b identical adult individuals of type b - "the quantity type" - with a low quality-parameter $\beta^b < \underline{\beta}$.[33] Since the quality parameter is transmitted without alteration within a dynasty, and since Proposition 1 implies that given the rate of technological progress parental type is the sole determinant of offspring education, it follows that in each

[33] As will become apparent, in order to assure that the differences in fertility across types is sufficiently small, the differences in β across types is assumed to be sufficiently small.

period t, the population size of generation t, L_t, consists of two homogenous groups of type a and b, whose size is L_t^a and L_t^b, respectively. That is, $L_t = L_t^b + L_t^a$.

Till period $t = -2$, the population of the world is homogeneous and it consists of type b individuals. In period $t = -2$, however, a very small fraction of the adult population gives birth to mutants of type a. In period $t = -1$, the mutants become adults individuals of type a whose parent are of type b[34] Finally, in all periods $t \geq 0$, all individuals of type a have parents who are of type a as well.

The optimal investment in child quality by members of each dynasty of type i is affected by their attitude towards child quality and the rate of technological progress.

Lemma 2 *Suppose that $\beta^b < \underline{\beta} < \beta^a$. Under A1, as depicted in Figure 2, investment in child quality in each dynasty of type i, $i = a, b$, is:*

$$e_t^a > 0 \qquad \forall t$$

$$e_t^b > 0 \qquad \textit{if and only if} \qquad g_t > \underline{g}(\beta^b) \equiv \underline{g}^b > 0$$

$$e_t^a > e_t^b \qquad \forall t$$

Proof. Follows from Lemma 1 and the definition of $\underline{\beta}$. □

The argument behind Lemma 2 is straightforward. For individuals of type a, $\beta^a > \underline{\beta}$, where $\underline{\beta}$ denotes the threshold level of the quality parameter above which individuals of generation t invest in the education of their offspring even if $g_{t+1} = 0$. Hence, it follows from the non-negativity of g_t that within a dynasty of type a investment in child quality, e_t^a, is strictly positive for all t. For individuals of type b, however, $\beta^b < \underline{\beta}$ and investment in child quality takes place if and only if the rate of technological change and hence the return to quality is sufficiently large. Furthermore, as follows from (6) $h_t^a > h_t^b$ for all t.

Let q_t denote the fraction of individuals of type a in generation t.

$$q_t \equiv L_t^a / L_t. \tag{12}$$

[34] The existence of a large number of types would not affect the qualitative analysis. The presence of two types of individuals simplifies the exposition considerably and permits the analysis of the effect of a single quality-parameter on the evolution of this complex three-dimensional system. Since the process of evolution is inherently associated with an improvement in the fitness and hence the evolutionary advantage of certain mutants, the underlying assumption is that when mutation starts affecting the economy in period 0, it introduces a type that at least temporary has an evolutionary advantage (i.e., a type with a parameter β that is closer to the optimal level relative to the pre-existing type - b)

The average level of education, e_t, as depicted in Figure 2, is therefore

$$e_t = q_t e^a(g_t) + (1 - q_t)e^b(g_t) \equiv e(g_t, q_t), \tag{13}$$

where as follows from Lemma 1, 2, and Assumption A2 $e_g(g_t, q_t) > 0$, $e_{gg}(g_t, q_t) < 0$, and $e_q(g_t, q_t) > 0$, $\forall g_t > 0$ and $\forall q_t > 0$. Hence, as depicted in Figure 2, the function $e(g_t, q_t)$ is increasing and piecewise strictly concave with respect to g_t.

The aggregate supply of efficiency units in period t, H_t, is

$$H_t = L_t^a f_t^a h_t^a + L_t^b f_t^b h_t^b = L_t[q_t f_t^a h_t^a + (1 - q_t) f_t^b h_t^b] \tag{14}$$

where f_t^i is the fraction of time devoted to labor force participation by an individual of type i. As follows from (8), noting that, $z_t^i = x_t^\alpha h^i(g_t)$ for $i = a, b$,

$$f_t^i = \begin{cases} 1 - \gamma & if \quad z_t^i \geq \tilde{z} \\ \tilde{c}/z_t^i & if \quad z_t^i \square \tilde{z} \end{cases} \equiv f^i(g_t, x_t). \tag{15}$$

where, as follows from (4) and Assumption A3, $f_x^i(g_t, x_t) < 0$ and $f_g^i(g_t, x_t) > 0$ for $z_t^i \square \tilde{z}$.

3 The Time Path of the Macroeconomic Variables

3.1 Technological Progress

Suppose that technological progress, g_{t+1}, that takes place from periods t to period $t+1$ depends upon the average quality (education) among the working generation in period t, e_t.

$$g_{t+1} \equiv \frac{A_{t+1} - A_t}{A_t} = \psi(e_t) \tag{16}$$

where $\psi'(e_t) > 0$ and $\psi''(e_t) < 0$ $\forall e_t > 0$ and $\psi(0) = 0$. Hence, the rate of technological progress between time t and $t+1$ is a positive, strictly increasing, strictly concave function of the average level of education of the working generation at time t.

The level of technology at time $t+1$, A_{t+1}, is therefore

$$A_{t+1} = [1 + g_{t+1}]A_t = [1 + \psi(e_t)]A_t, \tag{17}$$

where the technological level at time 0 is historically given at a level A_0.

The abstraction from the complementary role of the scale of the economy (i.e.,the size of the population) in the determination of technological progress is designed to sharpen the focus on the role of the evolutionary process in the demographic transition and modern growth.[35] As will become apparent, the focus on the role of the quality-composition of the labor force as the driving force of technological progress assures that natural selection is a necessary condition for the demographic transition and the take-off for modern growth.

Hence as follows from (13), (16), and Lemma 1 and 2, g_{t+1} is uniquely determined by g_t and q_t.

$$g_{t+1} = \psi(e(q_t, g_t)) \equiv g(g_t, q_t), \tag{18}$$

where $g_q(g_t, q_t) > 0$, $g_g(g_t, q_t) > 0$, and $g_{gg}(g_t, q_t) < 0$.

3.2 Population and Fertility Rates Across Types

The evolution of the population of type i over time is given by

$$L_{t+1}^i = n_t^i L_t^i, \tag{19}$$

where n_t^i is the number of children of each individual of type $i = a, b$, and L_t^i is the size of the population of type i in generation t, where L_0^i is given. Given that $g_{t+1} = g(g_t, q_t)$, it follows from (11) that

$$n_t^i = n^i(g_t, x_t, q_t), \qquad i = a, b. \tag{20}$$

The evolution of the working population over time is given by

$$L_{t+1} = n_t L_t, \tag{21}$$

where $L_t = L_t^b + L_t^a$, is the population size of generation t, and n_t is the average fertility rate in the population. That is,

$$n_t \equiv q_t n_t^a + (1 - q_t) n_t^b, \tag{22}$$

where as defined in (12), $q_t \equiv L_t^a / L_t$ is the fraction of adult individuals of type a in generation t (born to type a individuals).

[35]Evidence regarding the role of the scale of the economy in technological progress is mixed. While Kremer (1993) provides some supporting historical evidence, Jones (1995) argues that in the 20th century it appears that there is no scale effect.

The evolution of q_t, as follows from , (12), (18) (19) and (20) is therefore

$$q_{t+1} = \frac{n_t^a}{n_t} q_t \equiv q(g_t, x_t, q_t) \tag{23}$$

where $n_t^i = n^i(g_t, x_t, q_t), \;\; i = a, b.$

The analysis of the relationship between the economic environment and the evolutionary advantage of different types of individuals indicates that in the early Malthusian era, when humans merely struggle for survival, individuals of type a (i.e., individuals with a preference bias towards quality of offspring) have an evolutionary advantage over individual of type b. That is, the fraction of individuals of type a, q_t, rises in the population, despite their preference bias against the quantity of their offspring. However, once the economic environment improves sufficiently the evolutionary pressure weakens, the significance of quality for survival (fertility) declines, and type b individuals – the quantity type – gain the evolutionary advantage.

Lemma 3 *Under A1, for any given* $g_t \geq 0$*, as depicted in Figure 4, there exist a unique* $\breve{x}_t \in ([\tilde{c}/h^b(g_t)]^{1/\alpha}, [\tilde{z}/h^b(g_t)]^{1/\alpha}) \equiv \breve{x}(g_t; q)$ *such that* $\forall x_t > [\tilde{c}/h^b(g_t)]^{1/\alpha}$ *(i.e.,* $\forall z_t^b > \tilde{c}$*),*

$$n_t^a \begin{cases} > n_t^b & for \quad x_t < \breve{x}_t \\ = n_t^b & for \quad x_t = \breve{x}_t \\ < n_t^b & for \quad x_t > \breve{x}_t \end{cases}$$

Proof. As follows from (11) $n_t^a > n_t^b = 0$ for $x_t = [\tilde{c}/h^b(g_t)]^{1/\alpha}$ and $n_t^b > n_t^a$ for $x_t \geq [\tilde{z}/h^b(g_t)]^{1/\alpha}$. Hence, since $\forall x_t \in ([\tilde{c}/h^b(g_t)]^{1/\alpha}, [\tilde{z}/h^b(g_t)]^{1/\alpha})$ (i.e., for the range under which $\partial n^b(g_t, x_t; q)/\partial x_t > 0$)

$$\partial n^b(g_t, x_t; q)/\partial x_t > \partial n^a(g_t, x_t; q)/\partial x_t$$

(Noting that as follows from Lemma 2 $e_t^a > e_t^b \;\; \forall t > 0$) the lemma follows from the intermediate value theorem. □

Figure 4 depicts the fertility rates of the two types, n_t^b and n_t^a, as a function of effective resources per efficiency unit of labor x_t, given the rate of technological progress, $g_t \geq 0$. Initially effective resources per efficiency unit of labor are sufficiently low (less than $\breve{x}(g_t; q)$) and the fraction of individuals of type a in the population increases. However, as the level of effective resources per efficiency unit of labor increases sufficiently (i.e., $x_t > \breve{x}(g_t; q)$) and the Malthusian pressure relaxes, the rate of population growth among individuals of type b - the quantity type - overtakes the rate among type $a-$ the quality type.[36] It should be noted that as established in Proposition 1 the increase in the rate of technological progress that brings about the increase in effective resources generates initially an increase in fertility rates of both types of individuals, but ultimately, due to the substitution of quality for quantity a demographic transition takes place and fertility rates decline.[37]

The absolute magnitude of the fertility rates of the two types of individuals depends upon the rate of technological progress.

Lemma 4 *For g_t and q_t such that $g_{t+1} = g(g_t, q_t) \square \underline{g}^b$, there exists a unique level of effective resources per efficiency unit of labor, $\grave{x}(g_t, q) \in (0, [\tilde{z}/h^b(g_t)]^{1/\alpha})$, such that the fertility rate of type b individuals is at replacement level, i.e.,*

$$n^b(g_t, \grave{x}(g_t, q), q) = 1 \quad for \quad g(g_t, q_t) \square \underline{g}^b.$$

Proof. As follows from (11),

$$n_t^b \begin{cases} = 0 & \forall x_t \square [\tilde{c}/h^b(g_t)]^{1/\alpha} \\ > 1 & \forall x_t \geq [\tilde{z}/h^b(g_t)]^{1/\alpha} \end{cases} \quad \text{for} \quad g(g_t, q_t) \square \underline{g}^b$$

Hence, since n_t^b is continuous and monotonically increasing in x_t the lemma follows from the intermediate value theorem. □

Suppose that prior to occurrence of mutations in period $t = -2$, the economy is in a steady-state equilibrium where the rate of technological progress is 0. (Since the entire population is of type b, i.e., $q = 0$, as will become apparent, this implies that in some historical period the rate of technological progress was sufficiently small (i.e., $g_t < \bar{g}^U(0)$)). Furthermore, since n_t^b increases in x_t, and x_t decreases when $n_t^b > 1$ and increases when $n_t^b < 1$, it follows from Lemma 4 that in this steady-state equilibrium, fertility rate is precisely at replacement level, i.e., $n_t^b = 1$, and effective resources per efficiency unit of labor is $\grave{x}$.

Since the process of evolution is inherently associated with an improvement in the fitness and hence the evolutionary advantage of certain mutants, the underlying assumption is that

[36]It should be noted that fertility rates of type b individuals exceeds those of type a, when type b individuals are still constrained by subsistence consumption. However, for type a the constraint may not be binding. Figure 4 is drawn for the case in which the constraint is binding for both types.

[37]An increase in g_t shifts the curves $n^a(g_t, x_t; q)$ and $n^b(g_t, x_t; q)$ in Figure 4 rightward and downward.

when mutation starts affecting the economy in period 0, it introduces a type that at least temporary has an evolutionary advantage (i.e., a type with a parameter β that is closer to the optimal level, given the economic environment, relative to the pre-existing type - b).[38] Hence, it is assumed that the quality parameter of individuals of type a, β^a, generates an evolutionary advantage in the period in which the mutation become effective. Namely,

$$\breve{x}(0;0) > \grave{x}(0,0), \tag{A4}$$

where $\grave{x}(0,0) = \tilde{c}/[1-\tau^n])^{1/\alpha}$ as follows from (11). That is $x_{-2} < \breve{x}_{-2.}$ Since the size of the population of type a is assumed to be very small, it has a negligible affect on the size of x_0 and therefore in period 0, $x_0 < \breve{x}_{0.}$ Hence, as follows from Lemma 3 and 4,

$$n_0^a > n_0^b = 1. \tag{24}$$

Hence, in early stages of development the Malthusian pressure provides an evolutionary advantage to the quality type. The income of individuals of the quantity type is near subsistence and fertility rates are therefore near replacement level. In contrast, the wealthier, quality type, can afford higher fertility rates (of higher quality offspring). As technological progress brings about an increase in income, the Malthusian pressure relaxes, and the domination of wealth in fertility decisions diminishes. The inherent advantage of the quantity type in reproduction gradually dominates and fertility rates of the quantity type ultimately overtake those of the quality type.

3.3 Human Capital and Effective Resources

As follows from (14), the growth rate of efficiency units of labor, μ_{t+1}, is

$$\mu_{t+1} \equiv \frac{H_{t+1}}{H_t} - 1 = \frac{q_t n_t^a f_{t+1}^a h_{t+1}^a + (1-q_t) n_t^b f_{t+1}^b h_{t+1}^b}{q_t f_t^a h_t^a + (1-q_t) f_t^b h_t^b} - 1 \tag{25}$$

[38]Mutation occurs in period $t = -2$. A very small fraction of the adult population in period $t = -2$ gives birth to mutants whose quality parameter, β^a, is higher than that in the existing adult population. In period $t = -1$, the mutants are adults who make fertility decisions. Their income is identical to that of type b individuals but their fertility rate is nevertheless lower due to their higher preference for child quality. In period $t = 0$ the mutants are "regular" individuals of type a whose potential income is higher than type b individuals. Hence, mutation has a real affect on output only in period 0.

Lemma 5 *Under A1 and A3,* $\forall x_t > [\tilde{c}/h^b(g_t)]^{1/\alpha}$ *(i.e.,* $\forall z_t^b > \tilde{c}$*)*,[39]

$$\mu_{t+1} = \mu(g_t, x_t, q_t)$$

where

$$\mu_x(g_t, x_t, q_t) \begin{cases} > 0 \quad if \quad x_t < [\tilde{z}/h^b(g_t)]^{1/\alpha} \\ = 0 \qquad\qquad otherwise \end{cases}$$

$$\mu_q(g_t, x_t, q_t)\big|_{g_{t+1}=g_t} \gtreqless 0 \quad \text{if and only if} \quad n_t^a \gtreqless n_t^b$$

$$\mu_g(g_t, x_t, q_t) < 0 \qquad\qquad \forall z_t^b \geq \tilde{z}$$

Proof. Substituting (11) and (18) into (25), noting (15), $\mu_{t+1} = \mu(g_t, x_t, q_t)$ and the properties follow, noting Proposition 1. □

The evolution of effective resources per efficiency unit of labor, $x_t \equiv A_t X/H_t$, depends on the rate of technological progress and the growth rate of efficiency units of labor. As follows from (18) and (25)

$$x_{t+1} = \frac{1 + g_{t+1}}{1 + \mu_{t+1}} x_t \equiv x(g_t, x_t, q_t). \tag{26}$$

4 The Dynamical System

The development of the economy is characterized by the trajectory of output, population, technology, education, and human capital. The dynamic path of the economy, is fully determined by a sequence $\{x_t, g_t, q_t\}_{t=0}^{\infty}$ that satisfies (18), (23) and (26) in every period t and describes the time path of effective resources per efficiency unit of labor, x_t, the rate of technological progress, g_t, and the fraction, q_t, of individuals of type a (the quality type) in the adult population.

The geometrical analysis of this three dimensional dynamical system is more transparent, however, if the equation of motion $g_{t+1} = \psi(e(g_t, q_t)) \equiv g(g_t, q_t)$, is decomposed into the two equations $g_{t+1} = \psi(e_t)$ and $e_t = e(g_t, q_t)$. Hence the dynamical system analyzed is a *four* dimensional non-linear first-order autonomous system

[39] As discussed previously, this is the range in which individuals of type b (and hecne, since $z_t^a > z_t^b$, individuals of type a) do not become extinct.

$$\begin{cases} x_{t+1} = x(g_t, x_t, q_t); \\ q_{t+1} = q(g_t, x_t, q_t); \\ g_{t+1} = \psi(e_t); \\ e_t = e(g_t, q_t). \end{cases} \tag{27}$$

The analysis of the dynamical system is greatly simplified since, holding q_t constant, the joint evolution of e_t and g_t, is determined independently of x_t, and it is independent of whether the subsistence constraint is binding.

4.1 Conditional Dynamics of Technology and Education

The evolution of the rate of technological progress and education, conditional on holding q constant, is characterized by the sequence $\{g_t, e_t; q\}_{t=0}^{\infty}$ that satisfies in every period t the conditional two dimensional system

$$\begin{cases} e_t = e(g_t; q) \\ g_{t+1} = \psi(e_t). \end{cases} \tag{28}$$

Although the conditional dynamical sub-system $g_{t+1} = \psi(e(g_t; q)) \equiv g(g_t; q)$ is a one dimensional system (given q), the analysis is more revealing in the context of the joint evolution of the two state variables.

In light of the properties of the function $e_t = e(g_t; q)$ and $g_{t+1} = \psi(e_t)$, given by (13) and (16), it follows that in any time period this conditional dynamical sub-system may be characterized by one of the two *qualitatively* different configurations, which are depicted in Figure 3. The economy shifts endogenously from one configuration to another as q increases and the curve $e_t = e(g_t; q)$ shifts upward to account for the positive effect of an increase in q on e_t.

As will become apparent, in order to allow for the existence of a long-run steady-state with a positive growth rate it is necessary to assume that

$$\exists g > 0 \quad s.t. \qquad e(g; 0) > \psi^{-1}(g).^{40} \tag{A5}$$

[40] Alternatively, $\exists\, g > 0$ such that $g(g, 0) > g$.

That is, in Figure 3(a), for $q = 0$, there exist a positive rate of technological progress, such that the curve $e(g, 0)$ lies above the curve $\psi(e_t)$ in the plain (g_t, e_t).

Lemma 6 *Under A1, A2 and A5, as depicted in Figure 3(a), for* $q = 0$, *the conditional dynamical system (28) is characterized by two locally stable steady-sate equilibria:*

$$[\bar{g}^L(q), \bar{e}^L(q)] = [0, 0]$$

$$[\bar{g}^H(q), \bar{e}^H(q)] >> [\underline{g}^b, 0]$$

Proof. Follows from the properties of $e_t = e(g_t; q)$ and $g_{t+1} = \psi(e_t)$, given by (13) and (16), Assumption A8, Lemma 1, and Lemma 2. □

Lemma 7 *Under A1, A2 and A5, there exist a critical level* $\hat{q} \in (0, 1)$ *such that*

$$e(\underline{g}^b, \hat{q}) = \psi^{-1}(\underline{g}^b).$$

Proof. It follows from the properties of $e_t = e(g_t; q)$ and Lemma 6 that $e(\underline{g}^b; 1) > \psi^{-1}(\underline{g}^b)$ and $e(\underline{g}^b; 0) < \psi^{-1}(\underline{g}^b)$. Therefore, the lemma follows from the continuity of $e(g_t; q)$ in q. □

Corollary 1 *Under A1, A2 and A5, as depicted in Figures 3(a)-(c), the set of steady-state equilibria of the conditional dynamical system (28) changes qualitative as the value of* q *passes the threshold level* $\hat{q}$. *That is for all* $q < \hat{q}$ *the system is characterized by multiple locally stable steady-sate equilibria, whereas for all* $q > \hat{q}$ *by a unique globally stable steady-state equilibrium.*[41]

$$\begin{cases} [\bar{g}^L(q), \bar{e}^L(q)] \text{ where } \bar{g}^L(q) < \underline{g}^b \\ [\bar{g}^H(q), \bar{e}^H(q)] >> [\underline{g}^b, 0] \end{cases} \quad for \quad q < \hat{q}$$

$$[\bar{g}^H(q), \bar{e}^H(q)] >> [\underline{g}^b, 0] \qquad for \quad q \geq \hat{q}$$

where for $j = L, H$,

$$\partial[\bar{g}^j(q), \bar{e}^j(q)]/\partial q >> 0.$$

[41] Note that for $q = \hat{q}$, the system is characterized by multiple steady-state equilibria. However, only the upper one is locally stable.

Proof. Follows from Lemma 6 and 7. □

In the first configuration the fraction of type a individuals (i.e., those with high preference for quality) is relatively low (i.e., $q < \hat{q}$). As depicted in Figures 3(a) (for $q = 0$) and Figure 3(b) (for $0 < q < \hat{q}$), the economy is characterized by multiple locally stable steady-state equilibria A low steady-state equilibria $[\bar{g}^L(q), \bar{e}^L(q)]$ where $\bar{g}^L(q) < \underline{g}^b$ and therefore only individuals of type a invest in human capital, and a high steady-state equilibrium $[\bar{g}^H(q), \bar{e}^H(q)] >> [\underline{g}^b, 0]$ where both types of individuals invest in human capital. As the value of q increases the values of g and e in each of the two stable steady-state equilibria increase as well.

In the second configuration the fraction of type a individuals - the quality type - is relatively high (i.e., $q \geq \hat{q}$). As depicted in Figure 3(c) (for $q > \hat{q}$), the economy is characterized by a unique globally stable steady-state equilibrium $[\bar{g}^H(q), \bar{e}^H(q)] >> [\underline{g}^b, 0]$ where both types of individuals invest in human capital. As the value of q increases the values of g and e in the steady-state equilibrium increase as well.[42]

4.2 Conditional Dynamics of Technology and Effective Resources

The evolution of the rate of technological progress, g_t, and effective resources per efficiency unit of labor, x_t, for a given ratio of type a individuals, q, is characterized by the sequence $\{g_t, x_t; q\}_{t=0}^{\infty}$ that satisfies in every period t the conditional two dimensional system:

$$\begin{cases} g_{t+1} = g(g_t; q); \\ x_{t+1} = x(g_t, x_t; q). \end{cases} \tag{29}$$

The phase diagrams of this conditional dynamical system, depicted in Figures 5(a)-5(c), contain three elements: the Subsistence Consumption Frontier, which separates the regions in which the subsistence constraint is binding for at least one type of individuals from those where it is not binding for both types; the XX locus, which denotes the set of all pairs (g_t, x_t) for which effective resources per efficiency unit of labor are constant; and the GG locus, which denotes the set of all pairs for which the rate of technological progress is constant.

[42] In the knife-edge case in which $q = \hat{q}$, the steady-state equilibrium $[\bar{g}^H(q), \bar{e}^H(q)] >> [\underline{g}^b, 0]$ is only locally stable.

4.2.1 The CC Locus

The economy exits from the subsistence consumption when potential income, z_t^i exceeds the critical level $\tilde{z}$ for all type of individuals, $i = a, b$. Since $z_t^a > z_t^b$ for all t, it follows from (4) and (10) that the switch occurs when $z_t^b = \tilde{z}$.

Let the *Subsistence Consumption Frontier, CC*, be the set of all pairs (g_t, x_t) for which $z_t^b = \tilde{z}$.

$$CC \equiv \{(g_t, x_t) : z_t^b = \tilde{z}\}, \tag{30}$$

where $z_t^b = x_t^\alpha h^b(g_{t+1})$ and $\tilde{z} = \tilde{c}/(1-\gamma)$

Lemma 8 *Under A1 and A3, there exists a single-valued strictly increasing function*

$$x_t = (\tilde{c}/[(1-\gamma)h^b(g_t)])^{1/\alpha} \equiv x^{CC}(g_t),$$

such that for all $g_t \geq 0$,

$$(g_t, x^{CC}(g_t)) \in CC,$$

where,

$$x^{CC}(0) = (\tilde{c}/[1-\gamma])^{1/\alpha};$$

$$\partial x^{CC}(g_t)/\partial g_t > 0.$$

Proof. Follows from Assumption A1 and A3, noting that $h(0,0) = 1$ and $e^b(0) = 0$. □

Hence, as depicted in Figure 5, the CC Locus, is an upward sloping curve in the plain (g_t, x_t) with a positive vertical intercept.

4.2.2 The GG Locus

Let GG be the locus of all pairs (g_t, x_t) such that, for a given level of q, the rate of technological progress, g_t, is in a steady-state.

$$GG \equiv \{(g_t, x_t; q) : g_{t+1} = g_t\} \tag{31}$$

As follows from (18), along the GG locus, $g_{t+1} = \psi(e(g_t; q)) \equiv g(g_t; q) = g_t$. The GG locus is therefore not effected by the effective resources per efficiency unit of labor, x_t, and as depicted in Figures 5(a)-5(c) the GG Locus consists of vertical line(s) at the steady-state level(s) of g, derived in Lemma 6 and Corollary 1 and depicted in Figures 3(a)-3(c).

As follows from the previous analysis there are two qualitatively different configurations . For $q < \hat{q}$, as depicted in Figures 5(a) and 5(b) (and corresponding to Figures 3(a) and 3(b)), the GG Locus consists of three vertical lines at the steady state level of $g : \{\bar{g}^L(q), \bar{g}^U(q), \bar{g}^H(q)\}$. For $q > \hat{q}$, as depicted in figure 5(c) (and corresponding to Figure 3(c)) the GG Locus consists of a unique vertical line at the steady-state level of $\bar{g}^H(q)$.[43]

Hence as follows from the properties of (18), for $q < \hat{q}$

$$g_{t+1} - g_t \begin{cases} > 0 \quad if \quad g_t < \bar{g}^L(q) \quad or \quad g_t \in (\bar{g}^U(q), \bar{g}^H(q)), \\ = 0 \quad if \quad g_t \in \{\bar{g}^L(q), \bar{g}^U(q), \bar{g}^H(q)\} \\ < 0 \quad if \quad g_t \in (\bar{g}^L(q), \bar{g}^U(q)) \quad or \quad g_t > \bar{g}^H(q), \end{cases}$$

whereas for $q > \hat{q}$

$$g_{t+1} - g_t \begin{cases} > 0 \quad if \quad g_t < \bar{g}^H(q), \\ = 0 \quad if \quad g_t = \bar{g}^H(q), \\ < 0 \quad if \quad g_t > \bar{g}^H(q). \end{cases}$$

4.2.3 The XX Locus

Let XX be the locus of all pairs (g_t, x_t) such that, for a given level of q, the effective resources per efficiency unit of labor, x_t, is in a steady-state.

$$XX \equiv \{(g_t, x_t; q) : \; x_{t+1} = x_t\}. \tag{32}$$

As follows from (26), along the XX locus, $x_{t+1} = [(1 + g_{t+1})/(1 + \mu_{t+1})]x_t \equiv x(g_t, x_t; q) = x_t$. Hence, along the XX Locus the growth rate of efficiency units of labor, μ_t, and the rate of technological progress, g_t, are equal. Thus, as follows from (18) and Lemma 5 along the XX Locus,

$$\mu(g_t, x_t; q) = g(g_t; q). \tag{33}$$

[43]For the knife-edge case of $q = \hat{q}$, $\bar{g}^L(\hat{q}) = \bar{g}^U(\hat{q}) = \hat{g}^b$, and the GG locus consists of two vertical lines at the steady-state level of $g : \{\hat{g}^b, \bar{g}^H(\hat{q})\}$.

To simplify the exposition and to assure the existence of the XX locus it is further assumed that

$$\mu_g(g_t, x_t; q) \,\square\, 0;^{44}$$

$$\lim_{g_t \to \infty} \mu(g_t, x^{CC}(g_t); q) \,\square\, 0;^{45} \qquad \text{(A6)}$$

$$\mu(0, x^{CC}(0); q) > g(0; q);^{46}$$

Lemma 9 and Corollary 2 derive the properties of the XX locus.

Lemma 9 *Under Assumptions A3-A6, given q, there exists a critical level of the rate of technological progress, $\hat{g}(q) > 0$ such that the XX Locus in the plane (g_t, x_t) is:*

1. *vertical at $g_t = \hat{g}(q)$, where $\hat{g}'(q) < 0$, for all x_t above the Subsistence Consumption Frontier, i.e.,*

$$(\hat{g}(q), x_t) \in XX \qquad \forall x_t \geq x^{CC}(\hat{g}(q));$$

2. *represented by a strictly increasing single value function $x_t = x^{XX}(g_t; q) > 0$ over the interval $[0, \hat{g}(q))$, i.e.,*

$$(g_t, x^{XX}(g_t; q)) \in XX \qquad \forall g_t \in [0, \hat{g}(q));$$

3. *below the Subsistence Consumption Frontier over the interval $[0, \hat{g}(q))$, i.e.,*

$$x^{XX}(g_t; q) < x^{CC}(g_t; q) \qquad \forall g_t \in [0, \hat{g}(q));$$

4. *empty for $g_t > \hat{g}(q)$, i.e.,*

$$(g_t, x_t) \notin XX \qquad \forall g_t > \hat{g}(q).$$

Proof.

[44] A sufficient condition for the negativity of $\mu_g(x_t, g_t, q_t)$ is a sufficiently small value of $|\partial h^i(g_t)/\partial g_t|$.

[45] This assumption is consistent with $\mu_g(g_t, x_t; q) \,\square\, 0$, given the feasible range of μ, i.e., $\mu \geq -1$.

[46] This condition is satisfied if $g(0, q)$ is sufficiently small, since as follows from Lemma 4, $\mu > 0$ weakly above the Malthusian frontier for $g_t = g_{t+1} = 0$.

1. If the XX locus is non-empty weakly above the CC frontier it is necessarily vertical in this range, since as follows from Lemma 5 $\mu_x(g_t, x_t, q) = 0$ above CC. Hence it is sufficient to establish that there exist a unique value $g_t = \hat{g}(q)$ such that $(\hat{g}(q), x^{CC}(\hat{g}(q)) \in XX$. As follows from Assumption A6, $\mu(0, x^{CC}(0); q) > g(0; q)$ and $\lim_{g_t \to \infty} \mu(g, x^{CC}(0); q) < \lim_{g_t \to \infty} g(g; q)$. Hence, since $\mu(g_t, x_t; q)$ is monotonically decreasing in g_t and $g(g_t; q)$ is monotonically increasing in g_t there exist a unique value $g_t = \hat{g}(q)$ such that $(\hat{g}(q), x^{CC}(\hat{g}(q)) \in XX$. Since along the XX locus $\mu(g_t, x^{CC}(g_t); q) = g(g_t; q)$ it follows from the properties of these function as derived in (18) and Lemma 5 that $\hat{g}'(q) < 0$

2. Given the existence of a unique value $g_t = \hat{g}(q)$ such that $(\hat{g}(q), x^{CC}(\hat{g}(q)) \in XX$, the existence of $x_t = x^{XX}(g_t; q)$ follows continuity and the implicit function theorem, noting (33) and the positivity of $\mu_x(g_t, x_t, q)$ over the interval $[0, \hat{g}(q))$, as established in Lemma 5 In particular,

$$\partial x^{XX}(g_t; q)/\partial g_t = [g_g(g_t; q) - \mu_g(g_t, x_t, q)]/\mu_x(g_t, x_t, q) > 0 \quad \forall g_t \in [0, \hat{g}(q)).$$

   (Note that as established in Lemma 5 $\mu_x(g_t, x_t, q) = 0$ for $g_t = \hat{g}(q)$, and the verticality of the XX Locus follows.) Furthermore, since $\mu(0, 0; q) = -1 < g(0; q)$, it follows that the vertical intercept of the XX locus is strictly positive. In particular, $x^{XX}(0,0) = (\tilde{c}/[1 - \tau^n])^{1/\alpha}$.

3. Given the uniqueness of the value $g_t = \hat{g}(q)$ such that $(\hat{g}(q), x^{CC}(\hat{g}(q)) \in XX$, it follows that the XX locus and the CC frontier do not intersect over the interval $[0, \hat{g}(q))$. In addition, the XX locus is vertical above the CC frontier. Hence, the XX locus is below the CC frontier in the range $[0, \hat{g}(q))$. In particular, $x^{XX}(0,0) = (\tilde{c}/[1 - \tau^n])^{1/\alpha} < x^{CC}(0, 0) = (\tilde{c}/[1 - \gamma])^{1/\alpha}$ since $\gamma > \tau^n$.

4. Given the uniqueness of the value of $g_t = \hat{g}(q)$ such that $(\hat{g}(q), x^{CC}(\hat{g}(q)) \in XX$, it follows that if the XX locus exists over the interval $(\hat{g}(q), \infty)$ than it must lie below the CC frontier. However, since $\mu_x(g_t, x_t, q) > 0$, and since along the XX locus $\mu(g_t, x_t; q) = g(g_t; q)$ it follows that along the CC frontier, over the interval $(\hat{g}(q), \infty)$, $\mu(g_t, x_t; q) > g(g_t; q)$, in contradiction to the fact that over the interval $(\hat{g}(q), \infty)$, $\mu(g_t, x_t; q) < g(g_t; q)$, as follows from Assumption A6 and established in part 1. □

Hence, as depicted in Figure 5, the XX Locus has a positive vertical intercept at $g = 0$, it increases monotonically with g_t, as long as $g_t \in [0, \hat{g}(q))$, and it becomes vertical at $g_t = \hat{g}(q)$. Furthermore, as q increases the value of $\hat{g}(q)$ declines.

Corollary 2 *Given q, there exists a unique pair $g_t = \hat{g}(q)$ and $x_t = x^{XX}(\hat{g}(q), q)$ such that $\{g_t, x_t; q,\} \in XX \cap CC$.*

Proof. Follows from Lemma 9. □

As follows from the properties of (18) and (25)

$$x_{t+1} - x_t \begin{cases} > 0 \quad if \quad x_t < x^{XX}(g_t) \quad or \quad g_t > \hat{g}(q) \\ = 0 \quad if \quad x_t = x^{XX}(g_t) \\ < 0 \quad if \quad x_t > x^{XX}(g_t) \text{ and } g_t < \hat{g}(q) \end{cases} \tag{34}$$

4.2.4 Conditional Steady-State Equilibria

This subsection describes the properties of the conditional steady-state equilibria of the conditional dynamical system $\{g_t, x_t; q\}_{t=0}^{\infty}$ based on the Phase diagrams depicted in Figure 5(a)-(c).

In order to assure the existence of a long-run (unconditional) steady-state equilibrium with sustained economic growth, it is further assumed that[47]

$$\hat{g}(0) < \bar{g}^H(0). \tag{A7}$$

Hence, since $\bar{g}^H(q)$ increases in q (Corollary 1) and since $\hat{g}(q)$ decreases in q (Lemma 9) it follows that

$$\hat{g}(q) < \bar{g}^H(q) \qquad \forall q. \tag{35}$$

[47] As follows from (34) Assumption A7 hold *if and only if* for all x_t, $x_{t+1} = x(\bar{g}^H(0), x_t; 0) - x_t > 0$, i.e., (noting (26)), *if an only if*, for all x_t $\mu(\bar{g}^H(0), x_t; 0)$ □ $\bar{g}^H(0)$. As follows from (25) $\mu(\bar{g}^H(0), x_t; 0) = n_t^b - 1$. Hence it follows from (11) that Assumption A7 holds *if and only if* γ □ $[\bar{g}^H(0) + 1][\tau^n + \tau^e e^b(\bar{g}^H(0))]$. Hence, Assumption A7 holds for sufficiently (i) high preference for quality by individuals of type b, β^b (since e^b and hence, $\bar{g}^H(0)$ increase with β^b), (ii) high cost of child raising τ^n, and (iii) low weight for children relative to consumption in the utility function, γ.

Hence, as depicted in Figures 5(a)-(c), and as established in the lemma below, Assumption A7 and (35) assure that if the economy crosses the Subsistence Consumption Frontier and enters into the Modern Growth Regime it would not cross back to the Malthusian regions.[48]

Furthermore, in order to assure that the economy converges to the modern growth regime, as is apparent from Figures 3 (a)-(c), it is necessary that the value of q increases sufficiently so as to pass the critical level, $\hat{q}$. Hence, it is necessary to assure that the fraction of individuals of type a in the population increases as long as $q \in [0, \hat{q}]$ and $g_t \in [0, \underline{g}^b]$. Since $n_t^a > n_t^b$ as long as $x_t < \breve{x}_t$, it is therefore sufficient to assume that

$$x^{XX}(g_t; q) < \breve{x}(g_t; q) \quad \text{for } g_t \in [0, \underline{g}^b] \text{ and } q \in [0, \hat{q}]. \tag{A8}$$

Lemma 10 *Under A2-A6 and A8,*

$$\hat{g}(q) > \underline{g}^b \qquad \forall q \in [0, \hat{q}].$$

Proof. As follows from (11) and Proposition 1, $n_t^b > n_t^a$ weakly above the Subsistence Consumption Frontier, and therefore $\breve{x}(g_t; q) < x^{CC}(g_t)$ for all g_t and q. Hence, it follows from Assumption A8 that $x^{XX}(g_t; q) < x^{CC}(g_t)$ for $g_t \in [0, \underline{g}^b]$ and $q \in [0, \hat{q}]$. As established in Lemma 9, $x^{XX}(g_t; q) < x^{CC}(g_t; q) \;\; \forall g_t \in [0, \hat{g}(q))$. It follows therefore that $\hat{g}(q) > \underline{g}^b$. □

Thus, as long as the economy is in the range of a low rate of technological progress, $g_t < \underline{g}^b$, and hence type b individuals do not invest in the quality of their offspring the economy can not take-off from the Malthusian regime.

The set of steady-state equilibria of this dynamical system consists of a constant growth rate of the technological level, and a constant growth rate (possibly zero) of effective resources per efficiency unit of labor. Let, χ_t denote the growth rate of effective resource per worker. As follows from (26)

$$\chi_t \equiv \frac{x_{t+1} - x_t}{x_t} = \frac{g_{t+1} - \mu_{t+1}}{1 + \mu_{t+1}} \equiv \chi(g_t, x_t; q). \tag{36}$$

Lemma 11 *Under A1-A8 as depicted in Figures 5(a)-(c), the set of steady-state equilibria of the conditional dynamical system (*29*) changes qualitative as the value of q passes the threshold*

[48]The incorporation of some additional plausible factors into the analysis, such as environmental effect on preferences (i.e. learning and imitation of the quality type) or positive effect of the scale of the population on technological progress would prevent the decline in the growth rate of output per capita in the advanced stages of the evolution of the economy towards the (unconditional) long-run equilibrium.

level $\hat{q}$. That is for all $q < \hat{q}$ the system is characterized by multiple locally stable steady-sate equilibria, whereas for all $q > \hat{q}$ by a unique globally stable steady-state equilibrium.[49]

$$\begin{cases} \left\{ \begin{array}{l} [\bar{g}^L(q), \bar{\chi}^L(q)] \quad \textit{where } \bar{g}^L(q) < \underline{g}^b \textit{ and } \bar{\chi}^L(q) = 0 \\ [\bar{g}^H(q), \bar{\chi}^H(q)] \quad \textit{where } \bar{g}^H(q) > \underline{g}^b \textit{and } \bar{\chi}^H(q) > 0 \end{array} \right\} & \textit{for} \quad q < \hat{q} \\ [\bar{g}^H(q), \bar{\chi}^H(q)] \quad \textit{where } \bar{g}^H(q) > \underline{g}^b \textit{and } \bar{\chi}^H(q) > 0 & \textit{for} \quad q \geq \hat{q} \end{cases}$$

where for $j = L, H$,

$$\partial \bar{g}^j(q)/\partial q > 0;$$
$$\partial \bar{\chi}^L(q)/\partial q = 0;$$
$$\partial \bar{\chi}^H(q)/\partial q > 0$$

Proof. The lemma follows from the properties of the CC locus, the XX locus and the GG locus derived in Lemma 6, 8, and 9, and their relative position in the plain (g_t, x_t) as follows from Assumption A7, and Lemma 10. Since the dynamical system is discrete, the trajectories implied by the phase diagrams do not necessarily approximate the actual dynamic path, unless the state variables evolve monotonically over time. As shown in section 4.1 the evolution of g_t is monotonic, whereas the evolution and convergence of x_t may be oscillatory. Non-monotonicity may arise only if $g < \hat{g}$. Non-monotonicity in the evolution of x_t does not affect the qualitative description of the system.[50] The local stability of the steady-state equilibrium $(0, \overline{x}(g_t))$ can be derived formally. The eigenvalues of the Jacobian matrix of the conditional dynamical system evaluated at the conditional steady-state equilibrium are both smaller than one (in absolute value) under (A1)-(A3). □

Hence, in early stages of development, when the fraction of individuals of type a in the population, q, is sufficiently small, the conditional dynamical system, as depicted in Figure 5(a) and 5(b) in the space (g_t, x_t), is characterized by two locally stable steady-state equilibrium that are given by the point of intersection between the GG Locus and the XX Locus. However, since the initial levels of g and q are infinitesimally small, the economy converges to the Malthusian steady-state equilibrium $[\bar{g}^L(q), \bar{x}^L(q)]$.

[49] Note that for $q = \hat{q}$, the system is characterized by multiple steady-state equilibria. However, only the upper one is locally stable.

[50] Furthermore, if $\partial x(g_t, x_t; q)/\partial x_t > -1$ for q □ $\hat{q}$ the conditional dynamical system is locally non-oscillatory.

In later stages of development as q_t increases sufficiently, the Malthusian conditional steady-state equilibrium vanishes. The dynamical system as depicted in Figure 5(c) is characterized by a unique steady-state equilibrium where the growth rates of the level of technology and the level of effective resources per efficiency unit of labor is constant at a level $[\bar{g}^H(q), \bar{\chi}^H(q)] >> 0$.

5 The Evolution of Mankind and Long Run Growth

This section analyzes the relationship between the evolution of mankind and economic growth from the emergence of the human spices. The analysis demonstrates that the inherent evolutionary pressure that is associated with the Malthusian equilibrium, had brought about the transition from Malthusian stagnation to sustained growth. The Malthusian pressure, via natural selection, increases the representation of individuals with a child-quality bias in the population, rising individuals' average quality and inducing higher rate of technological progress that ultimately brings about the evolution from Malthusian stagnation via the demographic transition to sustained growth.

The derivation of this long transition is based on the analysis of the motion of the conditional dynamical systems within each regime and the transition between the different regimes as the proportion of individuals with a high preference for child quality in the population evolves. This motion is reflected by two sequences of phase diagrams presented in Figures 3(a)-3(c) and 5(a)-5(c) that depict the changes in the evolution of $\{g_t, e_t\}$ and $\{g_t, x_t\}$, respectively, as the value of q_t evolves in the process of development.

In early periods, the population of the world consists of homogeneous individuals of the quantity type - type b - who care about the quality and the quantity of their children. The fraction of individuals of the quality type - type a - who places a higher weight on the quality of their children is zero (i.e., $q = 0$). Given the initial conditions the economy is therefore in a steady-state equilibrium where the rate of technological progress is zero (i.e., $g_t = 0$) and the average quality level in the population is zero (i.e., $e_t = 0$). Namely, as follows from (16) and Lemma 2, parents of type b have no incentive to raise quality children when the rate of technological progress is zero, whereas the rate of technological progress is zero when the average quality of the population is zero. Hence, as depicted in Figure 3(a) in the plain (g_t, e_t) the economy is in a locally stable steady-state equilibrium $[\bar{g}^L(0), \bar{e}^L(0)] = [0, 0]$. The level of effective resources and hence the rate of population growth is derived from the phase diagram depicted in Figure 5(a) in the plain (g_t, x_t). The economy is in a locally stable Malthusian steady-state equilibrium $[\bar{g}^L(0), \bar{x}^L(0)]$ where effective resources are constant at a level $\bar{x}^L(0) = \grave{x}(0, 0) > 0$, the level of human capital is constant, and hence, output per capita is constant as well. In this steady-state equilibrium the population is constant, and fertility rate is therefore at replacement level, i.e., $n_t^b = 1$.[51] Furthermore, (small) shocks to population or resources would be undone in a classic Malthusian fashion.

Mutation introduces a very small number of individuals of type a -"the quality type"- who places higher weight on the quality of their children.[52] Subsequently, in every period the economy consists of two types of individuals: individuals of type a - the "quality type" - with a higher weight for quality, and individuals of type b - the "quantity type" - with a lower weight for quality.

In the initial periods after mutation affects the economy the fraction of individuals of the quality type is sufficiently small, (i.e., $q_t < \hat{q}$). As depicted in Figure 3(b) in the space (g_t, e_t), for a given level of q, the economy is in the vicinity of a conditional locally stable steady-state equilibrium $[\bar{g}^L(q), \bar{e}^L(q)]$ where $\bar{g}^L(q) < \underline{g}^b$. As established in Lemma 2, the quality chosen by type b individuals is $e_t^b = 0$, the quality chosen by type a individuals is $e_t^a > 0$, and the average level of education, e_t, is therefore positive but small (i.e., $g_{t+1} = \psi(e_t) < \underline{g}^b$) since the fraction of individuals of type a is small. Furthermore, as depicted in Figure 5(b) in the space (g_t, x_t), this conditional locally stable steady-state equilibrium corresponds to a locally stable conditional Malthusian steady-state equilibrium, $[\bar{g}^L(q), \bar{x}^L(q)]$.where $\bar{g}^L(q) < \underline{g}^b$.

The analysis of the relationship between the economic environment and the evolutionary

[51]Since $\bar{g}^L(0) = 0$, and since $\bar{x}^L(0)$ is constant, it follows from $x_t \equiv A_t X / H_t$ and (14) that the population is constant and fertility rate is therefore at replacement level, i.e., $n_t^b = 1$. Furthermore, as follows from Lemma 4 $\bar{x}^L(0) = \grave{x}(0, 0) > 0$.

[52]This is a simplifying assumption that is designed to capture a sequence of mutations which result in a gradual increase in the variance in the distribution of the quality parameter. This process therefore has for a long period no effect on the quality composition of the population, since in the absence of technological progress there is a large range of $0 < \beta$ □ $\underline{\beta}$ for which individuals choose no investment in child quality. Ultimately mutation increases the variance sufficiently and individuals of type a - who invest in quality even in the absence of technological change - emerge. Clearly, the existence of heterogeneity of types throughout human history would not affect the qualitative analysis as long as the fraction of the quality type is initially small. The focus on two types of individuals simplifies the exposition considerably and permits the analysis of the evolution of this complex three-dimensional system.

advantage of different types of individuals indicates that in this early Malthusian era, when humans merely struggle for survival, individuals of type a (i.e., individuals with a preference bias towards quality of offspring) have an evolutionary advantage over individual of type b. That is, the fraction of individuals of type a rises in the population, despite their preference bias against the quantity of their offspring. Hence, in early stages of development the Malthusian pressure provides an evolutionary advantage to the quality type. The income of individuals of the quantity type is near subsistence and fertility rates are therefore near replacement level. In contrast, the wealthier, quality type, can afford higher fertility rates (of higher quality offspring). As follows from Assumption A8 and Lemma 3, $n_t^a > n_t^b$ for all $q_t < \hat{q}$, and hence the fraction of individuals of the quality type in the population, q_t increases monotonically over this Malthusian regime. As q_t increases the locus $e(g_t, q_t)$ in Figure 3(b) shifts upward and the corresponding conditional steady-state equilibrium reflects higher rate of technological progress along with higher average quality.

Eventually as q_t crosses the threshold level $\hat{q}$, the conditional dynamical system changes qualitatively. The $e(g_t, q_t)$ locus in Figure 3(b) shifts sufficiently upward so as to eliminate the lower intersection with the locus $g_{t+1} = \psi(e_t)$, and the loci GG^L and GG^U depicted in Figure 5(b) vanishes, whereas the GG^H locus shifts rightward and the XX locus above the Subsistence Consumption Frontier shifts leftward. As depicted in Figures 3(c) and 5(c) the Malthusian conditional steady-state equilibrium vanishes and the economy is no longer trap in the vicinity of this equilibrium. The economy converges gradually to a unique globally stable conditional steady-state equilibrium $[\bar{g}^H(q), \bar{e}^H(q), \bar{\chi}^H(q)] >> [\underline{g}^b, 0, 0]$ where both types of individuals invest in human capital, the rate of technological progress is high, and the growth rate of effective resources per efficiency unit of labor is positive. Once the rate of technological progress exceeds $\underline{g}^b$ - the threshold level of the rate of technological progress above which individuals of type b start investing in the quality of their children - the growth rate of the average level of education increases and consequently there is an acceleration in the rate of technological progress that may be associated with the Industrial Revolution. The positive feedback between the rate of technological progress and the level of education reinforces the growth process, the economy ultimately crosses the Subsistence Consumption Frontier, setting the stage for a demographic transition in which the rate of population growth declines and the average level of education increases.[53] The economy converges to the unique, stable, conditional steady state equilibrium above the Subsistence Consumption Frontier with a positive growth rate of output per worker.[54]

Technological progress has two effects on the evolution of population, as shown in Proposition 1. First, by inducing parents to give their children more education, technological progress will *ceteris paribus* lower the rate of population growth. But, second, by raising potential income, technological progress will increase the fraction of time that parents devote to raising children. Initially, while the economy is in the Malthusian region of Figure 5(b), the effect of technology on the parent's budget constraint will dominate, and so the growth rate of the population will increase. As the economy eventually crosses the Subsistence Consumption Frontier further improvements in technology no longer have the effect of changing the amount of time devoted to child-rearing. Faster technological change therefore raises the quality of children while reducing their number.

During the transition from the Malthusian to the Modern growth regime, once the economic environment improves sufficiently the evolutionary pressure weakens, the significance of quality for survival (fertility) declines, and type b individuals – the quantity type – gain the evolutionary advantage. Namely, as technological progress brings about an increase in income, the Malthusian pressure relaxes, and the domination of wealth in fertility decisions diminishes The inherent advantage of the quantity type in reproduction gradually dominates and fertility rates of the quantity type ultimately overtake those of the quality type (i.e., as the level of effective resources exceeds $\breve{x}$). Hence, the fraction of type a individuals, q_t, starts declining as the economy approaches the Subsistence Consumption Frontier. The model predicts therefore that the long run equilibrium is characterized by a complete domination of the quantity type (i.e., $q = 0$). Nevertheless, the growth rate of output per worker remains positive, although at a lower level than the one existed in the peak of the transition. As the level of q declined below the threshold level $\hat{q}$ the conditional dynamical system that describes the economy is

[53] Since differences in β across types are sufficiently small, the rates of change in q is sufficiently small, and a demographic transition and the increase in the average level of education in each population type implies the same pattern for the population as a whole (i.e., the decline in q, once the economy crosses the Malthusian frontier, is only a partially offsetting factor).

[54] It should be noted that once the fraction of individuals of the quality type exceeds $\hat{q}$ and therefore $g_t > \underline{g}^b$, the demographic transition occurs regardless of the evolutionary process.

once again characterized by multiple locally stable steady-state equilibria, as depicted in Figures 3(a),3(b),5(a), and 5(b). However unlike the situation in early stages of development, the position of the economy prior to the decline in q_t assures that the economy converges to the high steady-state equilibrium. The incorporation of some additional plausible factors into the analysis, such as environmental effect on preferences (i.e. learning and imitation of the quality type either in the Malthusian regime when the evolutionary pressure is binding or later) would permit heterogeneity of types in the long run. Furthermore, the incorporation of a positive effect of the scale of the population, (given quality) on the rate of technological progress might prevent the decline in the growth rate of output per capita in the advanced stages of the evolution of the economy towards the (unconditional) long-run equilibrium.

Finally, fertility differential across income groups evolves non-monotonically in the process of development. As depicted in Figure 4, in any period within the Malthusian Regime (i.e., as long as $g_t \; \square \; \underline{g}^b$ and therefore $x_t < \breve{x}$), fertility rates among richer individuals are predicted to be higher than those among poorer individuals, whereas in any period within the Modern Growth Regime (i.e., once $x_t \geq x^{CC}(g_t)$ and therefore $x_t > \breve{x}$) fertility rates among richer individuals are predicted to be lower than those among poorer individuals. Hence, in the course of the transition from the Malthusian Regime to the Modern Growth Regime the *cross section* relationship between income and fertility is reversed. In the Malthusian Regime there is a positive cross section correlation between income and fertility rates whereas in the Modern Growth Regime this cross section correlation is negative.

5.1 The Composition of Population and Failed Take-off Attempts

The analysis suggests that the interaction between the composition of the population and the rate of technological progress is the critical factor that determines the timing of the transition from stagnation to growth. In particular, the theory indicates that waves of rapid technological progress in the Pre-Industrial Revolution era had not generated a sustainable economic growth due to the shortage of individuals of the quality type in the population, whereas sustained economic growth in the post Industrial revolution era may be attributed to the presence of a sufficiently high fraction of individuals of the quality type in the population.

As depicted in Figure 5(a) and 5(b), if the fraction of individuals of the quality type is low, the economy is characterized by multiple steady-state equilibria. Two locally stable equilibria: a Malthusian steady-sate equilibrium where output per-capita is constant near a subsistence level of consumption and a modern growth steady-state equilibrium where a positive growth rate of output per capita is sustainable, as well as an unstable intermediate steady-state equilibrium.

Initial conditions places the economy in the vicinity of the Malthusian steady-state equilibrium. However, a sufficiently large technological shock would place the economy on a trajectory that leads to sustained growth. The composition of the population determines the effectiveness of a technological shock. The smaller is the fraction of individuals of the quality type in the population the larger is the necessary size of the shock in order to generate a sustained take-off form Malthusian stagnation. As the fraction of the quality type in the population increases (i.e., q_t rises) the distance between the loci GG^L and GG^U (depicted in Figure 5(b)) narrows and the necessary jump in the rate of technological progress in order to facilitate a sustained take-off decreases. Ultimately, as depicted in Figure 5(c) once q crosses the threshold level $\hat{q}$, the dynamical system changes qualitative. It is characterized by a unique globally stable steady-state equilibrium with sustained economic growth and the transition from Malthusian stagnation occurs without a need for a technological shock.

The analysis suggests therefore that non-sustainable growth episodes during the pre-Industrial Revolution period may be attributed to the presence of a relatively small fraction of individuals of the quality type in the population - a population that would have invested sufficiently in education in response to the change in the technological environment and would have therefore allowed this rapid change in technology to be sustained.[55] Furthermore, one may meaningfully argue that given the finiteness of a technological leap, an adverse composition of the population could have virtually prevented a sustained take-off from a Malthusian steady-state. Unlike the non-successful take-off attempts during the Greco-Roman period, the paper argues that, the successful take-off during the Industrial Revolution that has been attributed largely to the acceleration in the pace of technological progress, is at least partly due to the

[55]The effect of non sustainable technological advance on output growth would vanish gradually. It would generate an increase in the average human capital of the population, but at a level that would sustain only slower technological progress. This lower rate, however, would not sustain the return to human capital. The average human capital in the population would decline, leading to a decline in the rate of technological change that would ultimately end in a state of stagnation.

gradual evolution of the composition of the population that generated a sufficiently large mass of quality type individuals in the eve of the industrial revolution. This compositional change have allowed the pace of technological progress to be sustained by generating an impressive increase in the average level of education.[56]

6 Concluding Remarks

This research develops an evolutionary growth theory that captures the interplay between the evolution of mankind and economic growth since the emergence of the human species. This unified theory encompasses the observed intricate evolution of population, technology and income per capita in the long transition from an epoch of Malthusian stagnation to sustained economic growth. The theory suggests that the prolonged economic stagnation prior to the transition to sustained growth stimulated natural selection that shaped the evolution of the human species, whereas the evolution of the human species was the catalyst of the take-off from an epoch of stagnation through a demographic transition to sustained growth. Consistently with existing evidence, the theory argues that along the Malthusian era technology evolved rather slowly and population growth prevented sustained rise in income per capita. Human beings, like other species, have confronted the basic trade-off between offspring's quality and quantity in their implicit Darwinian survival strategies.[57] Although quantity-biased preferences had a positive direct effect on fertility rates, it had adversely affected the quality of offspring, their fitness, and hence their fertility rates. The inherent evolutionary pressure in the Malthusian era in which humans had limited resources for child rearing, generated an evolutionary advantage to quality-biased preferences. Natural selection therefore increased the quality of the population inducing faster technological progress that had brought about the take-off from the stagnation era and a demographic transition, that has paved the way to sustained economic growth.

The theory focuses on the change in the composition of types within the Homo Sapiens (i.e., variants within the species) rather than the more dramatic evolution from the Homo Erectus to the Homo Sapiens, for instance. Namely, the theory focuses on the evolution in

[56]For example, the average number of years of schooling in England and Wales rose from 2.3 for the cohort born between 1801 and 1805 to 5.2 for the cohort born 1852-56 and 9.1 for the cohort born 1897-1906. (Robert C. O. Matthews, Charles H. Feinstein, and John C. Odling-Smee, 1982).

[57]In other species this trade-off is implicit in their biological mechanism.

the composition of types within a population that has only a modest variety in genetic traits across types. The theory abstracts therefore from the evolution in the size of the human brain, focusing on the evolution of preferences within the Homo Sapiens.[58] Evidence regarding evolutionary process in nature suggests that evolutionary processes in the composition of types is rather rapid.[59]

Unlike previous unified theories, the presence of heterogeneity in the proposed theory generates predictions regarding the evolution of fertility across individuals within a time period, as well as over time. The theory predicts that fertility differential across income groups evolves non-monotonically in the process of development. In any period within the Malthusian Regime fertility rates among richer individuals are predicted to be higher than those among poorer individuals, whereas in any period within the Modern Growth Regime fertility rates among richer individuals are predicted to be lower than those among poorer individuals. Hence, in the course of the transition from the Malthusian Regime to the Modern Growth Regime the cross-section relationship between income and fertility is reversed. In the Malthusian Regime there is a positive cross-section correlation between income and fertility rates whereas in the Modern Growth Regime this cross-section correlation is negative. This prediction is consistent with evidence for the existence of a hump shaped cross-section relationship between fertility and income per-capita (e.g., Ronald Lee, 1987 and George Boyer, 1989, Livi-Bacci, 1997).

The theory suggests that the interaction between the composition of the population and the rate of technological progress determines the timing of the transition from stagnation to growth. In particular, the theory indicates that waves of rapid technological progress in the Pre-Industrial Revolution era (e.g., during the Greco-Roman period) had not generated a sustained economic growth due to the shortage of individuals of the quality type in the population.

[58]In contrast to the clear evolutionary trade-off that is introduced by the choice between quality and quantity of offspring, a focus on the evolution in brain size appears somewhat less interesting from an economic viewpoint. In particular, from the Neolithic period and till the demographic transition it appears that higher intelligence had no obvious evolutionary trade-off; Higher intelligence had been associate with higher potential income and had generated an absolute evolutionary advantage. In a sequel to this paper, Galor and Moav (2000) develop a unified theory that focuses on the evolution of intelligence and the origin of economic growth. As is established in this study, a quality-quantity trade-off is a necessary condition for the demographic transition.

[59]For instance, H. B. D. Kettlewell (1973) 's field experiments on industrial melanism in the peppered moth, Biston Betularia, has shown that given its typical background of a white tree trunk this moth has been white with a small population of black mutants. However, in areas in which industrial black carbon changed the background color the entire population turned black within a short time period. In contrast, the evolution from the Homo Erectus to Homo Sapiens, in which brain size nearly doubled, had taken more than 1 million years.

Although the return to quality increased temporarily, the level of human capital that was generated by the response of the existing population was not sufficient to support sustained technological progress and economic growth. In contrast, the era of sustained economic growth in the aftermath of the Industrial Revolution may be attributed to the presence of a sufficiently large fraction of quality type individuals in the population whose vigorous response to the rise in the return to human capital has supported sustained technological progress and economic growth.[60]

[60]One should not be concerned about the possibility that the quality-type would reach a complete domination very early in the evolution of mankind. Two explanations may be offered: (i) The spirit of the formal argument is that sequence of mutations which result in a gradual increase in the variance in the distribution of the quality parameter have no effect on the composition of the population for a long period, since in the absence of technological progress there is a large range of the quality parameter for which individuals choose no investment in child quality. Ultimately mutations increase the variance sufficiently and individuals who invest in quality even in the absence of technological change emerge. (ii) In earlier period people lived in "protocommunistic society". Child rearing was a common responsibility and hence prevented the manifestation of the potential evolutionary advantage of the quality type. An increase in population density, gave rise to further division of labor, stronger family structure and intergenerational income link. The quality type therefore gained the evolutionary advantage only in this late stage. This suggests testable predictions about the positive role of the family structure and intergenerational links in the emergence from the epoch of stagnation and the onset of the demographic transitions.

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Figure 1. Preferences, Constraints and Income Expansion Path

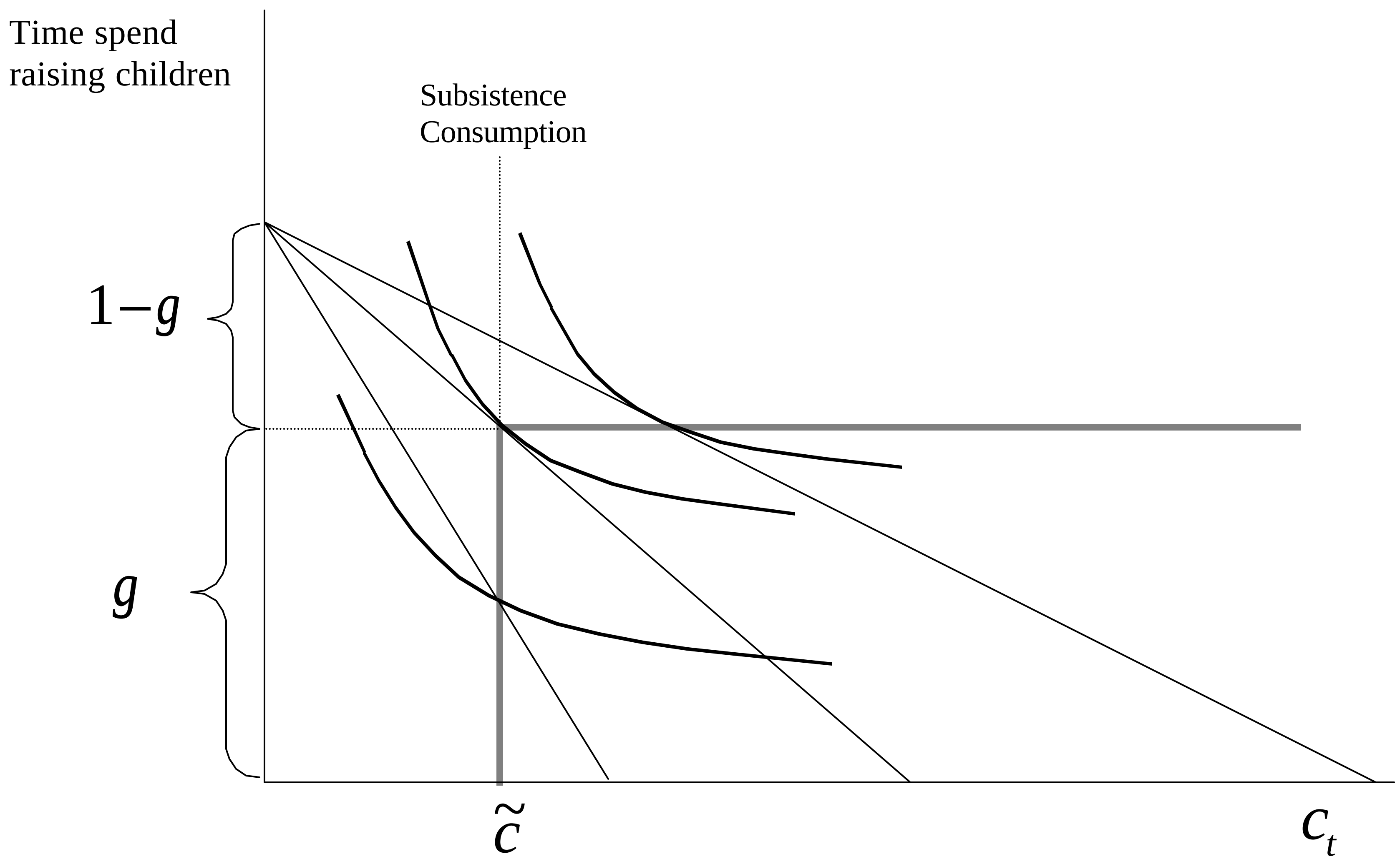

Figure 2. The Effect of the Rate of Technological Progress on the Average Quality

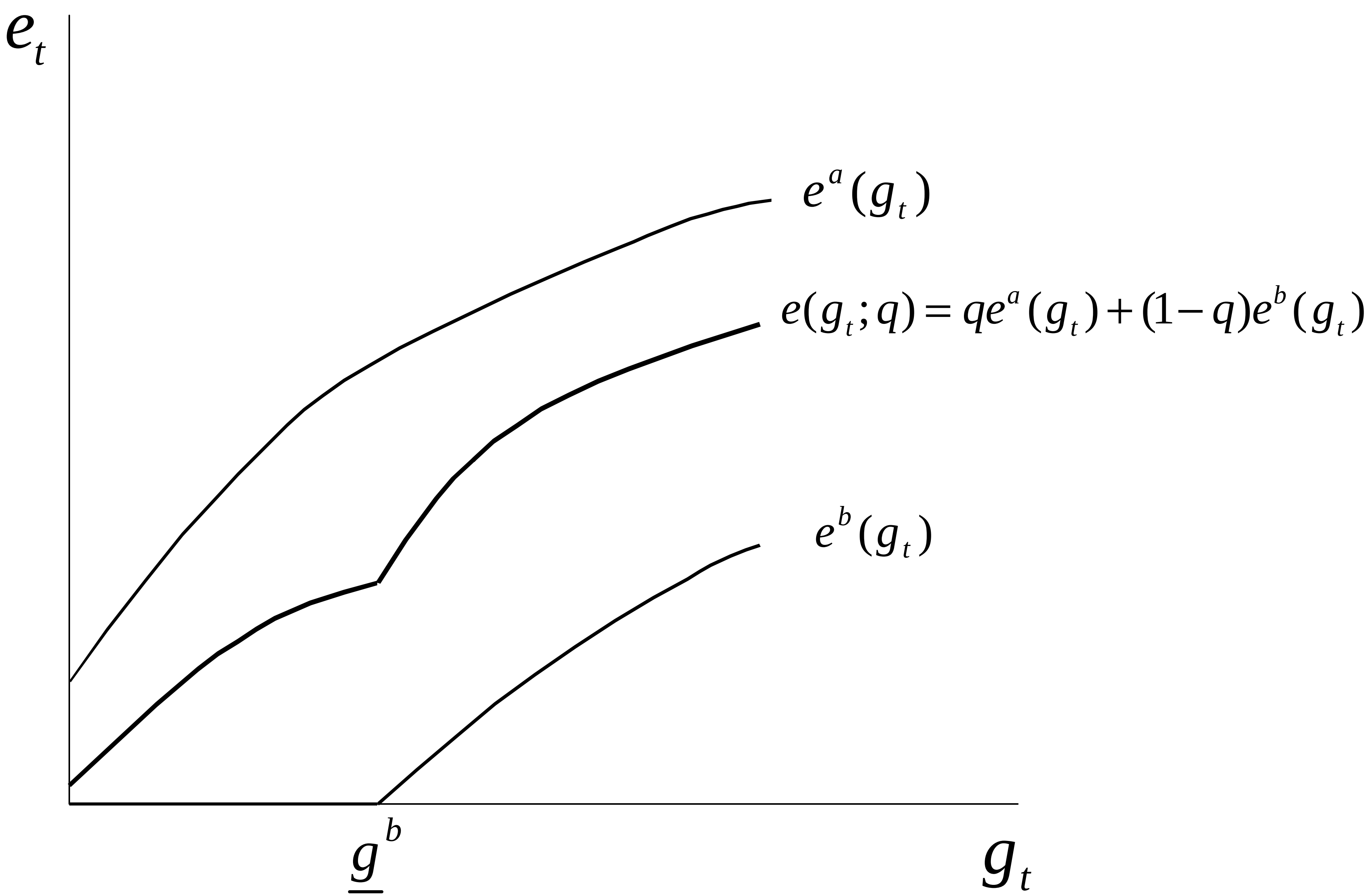

Figure 3(a). The Evolution of Education
and Technological Progress

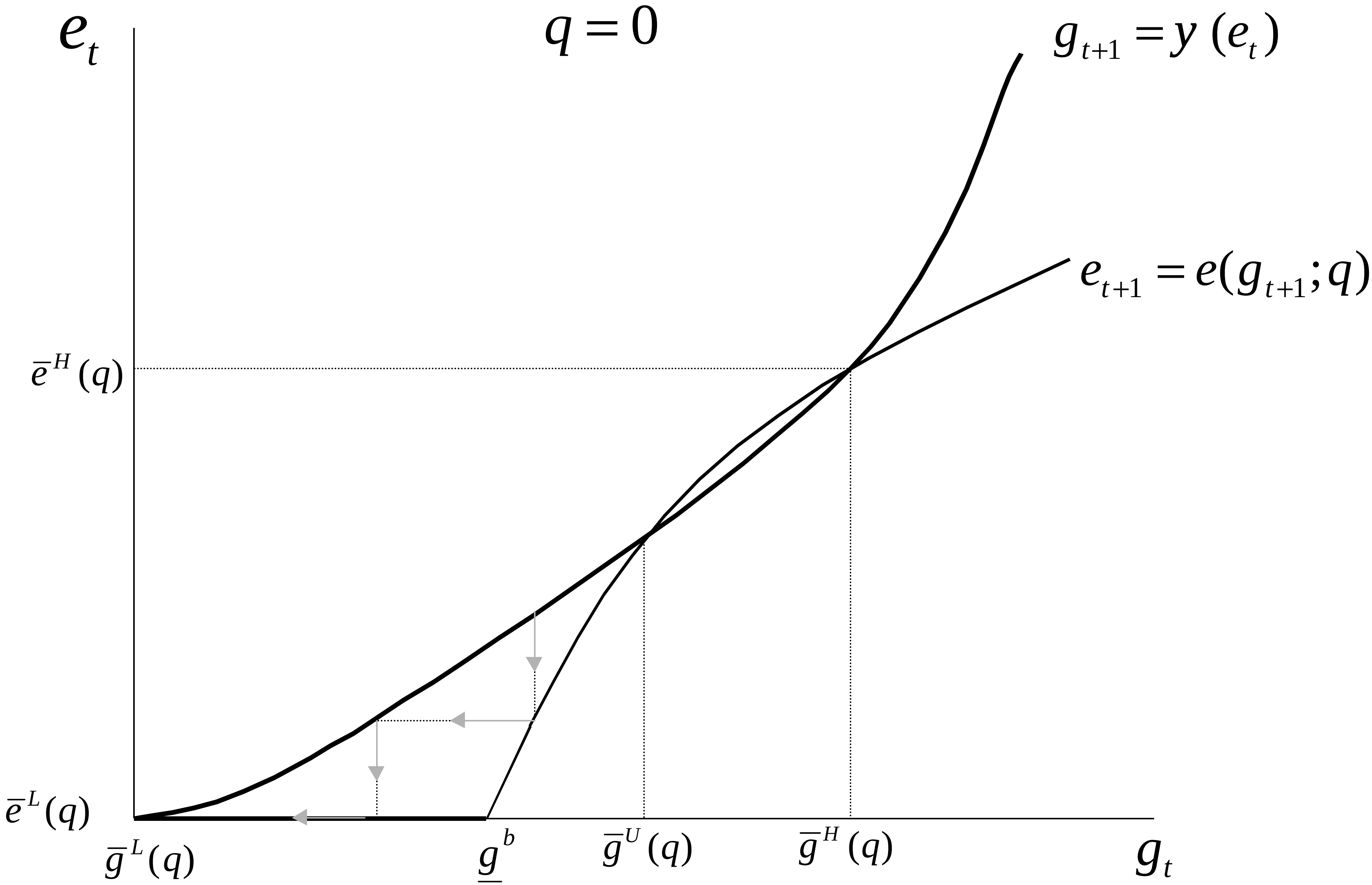

Figure 3(b). The Evolution of Education and Technological Progress.

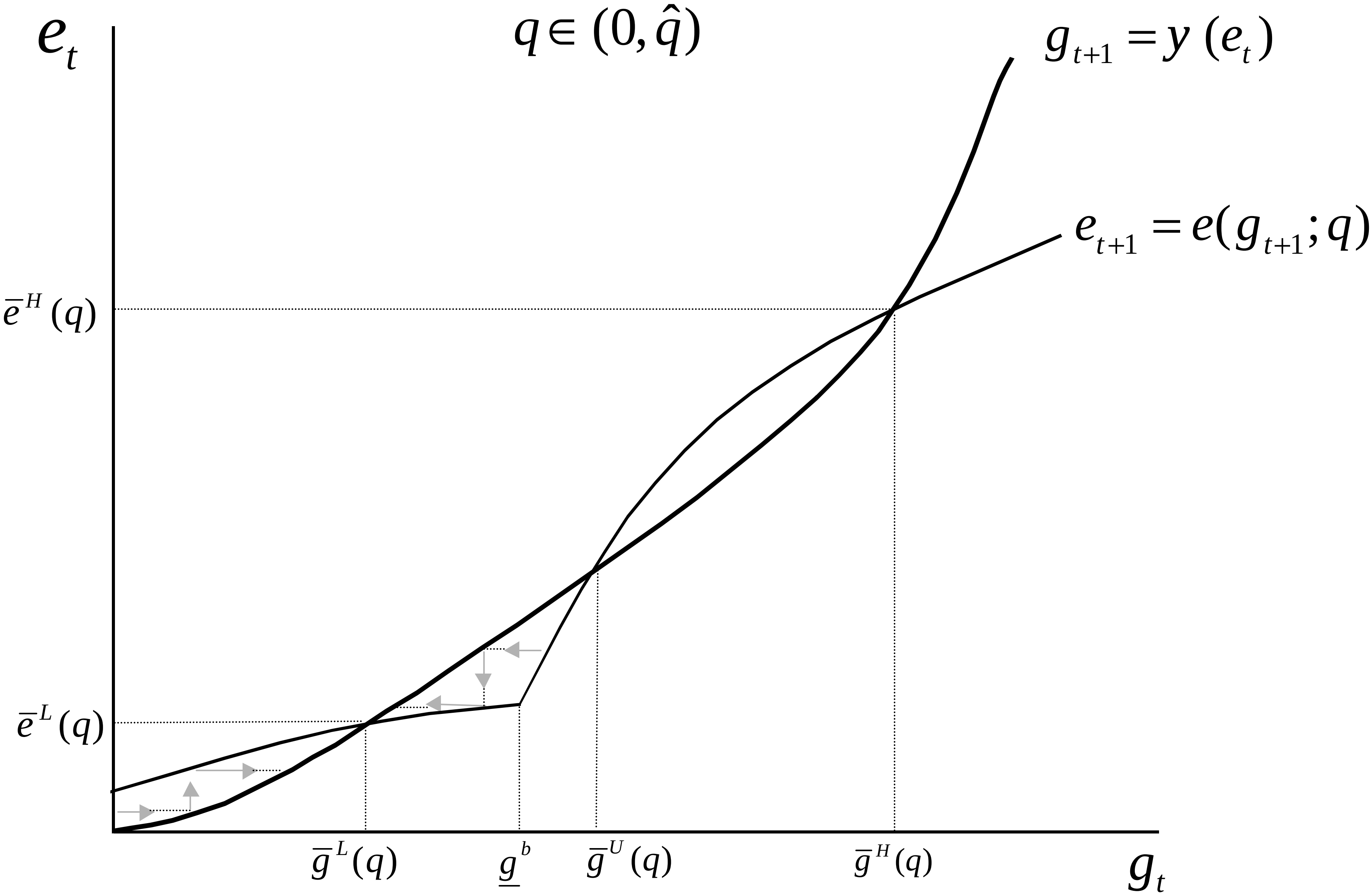

Figure 3(c). The evolution of education
and technological progress.

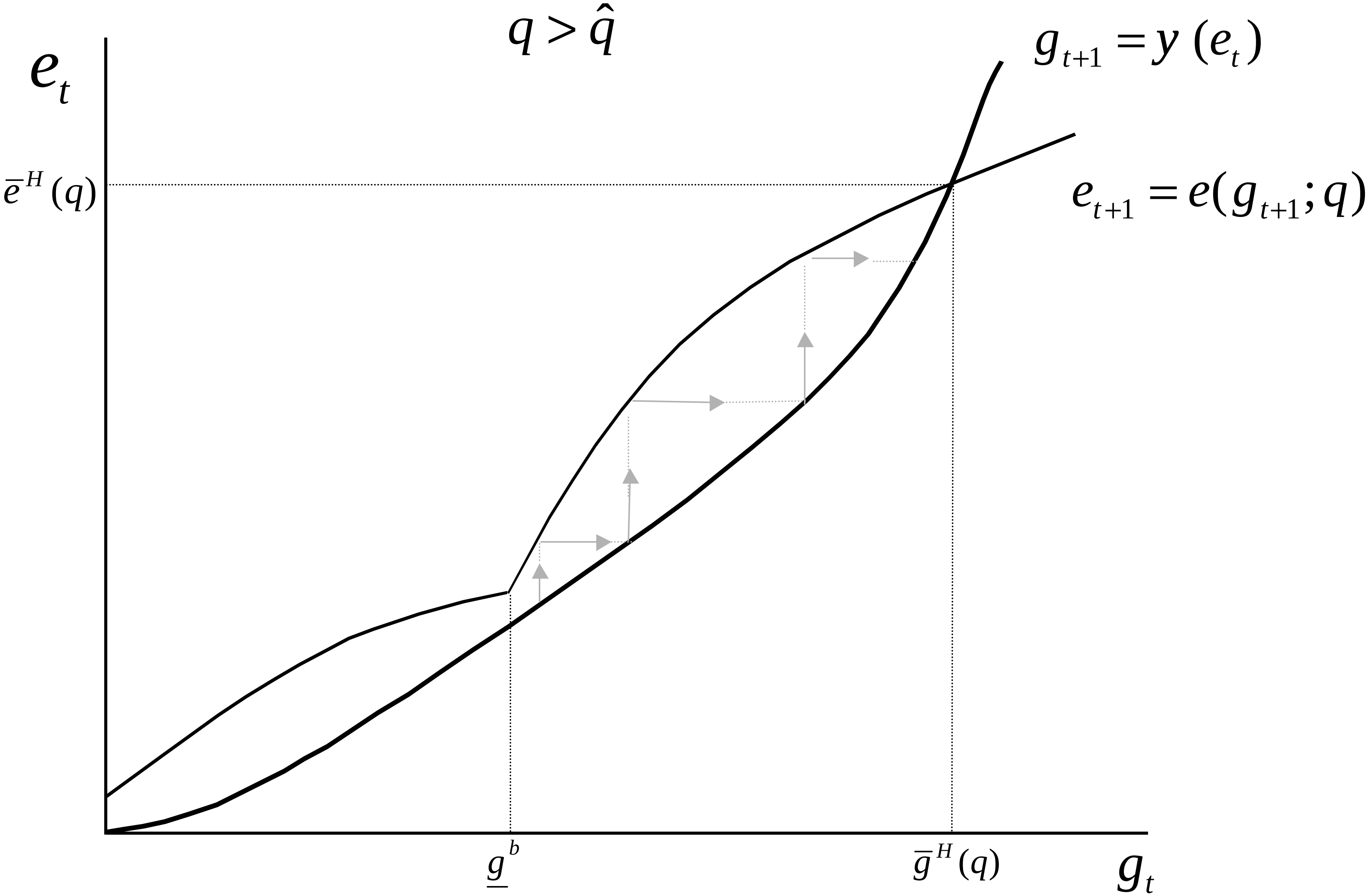

Figure 4. Fertility Rates Across Types

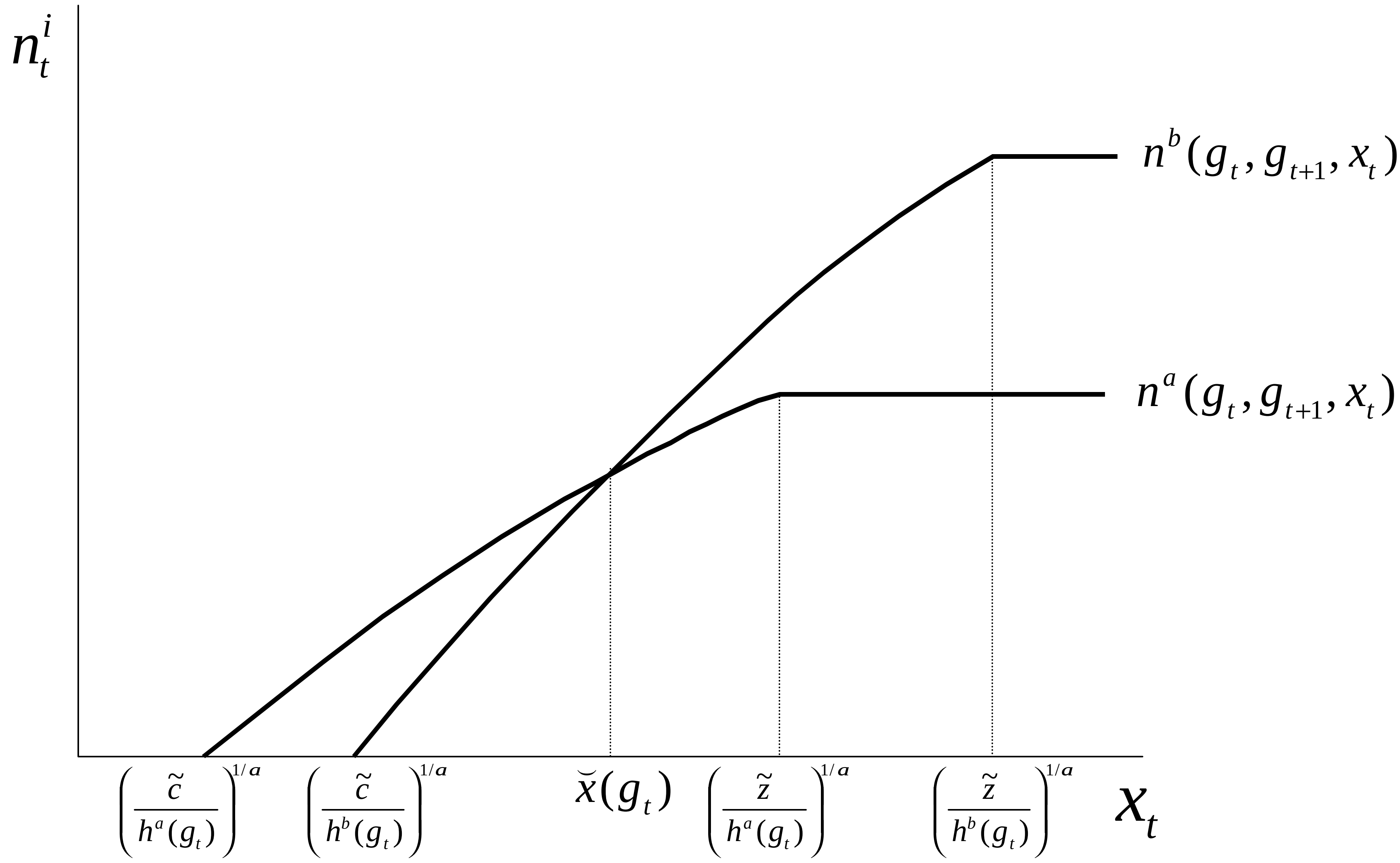

Figure 5(a). The Evolution of Technological Progress and Effective Resources

$q = 0$

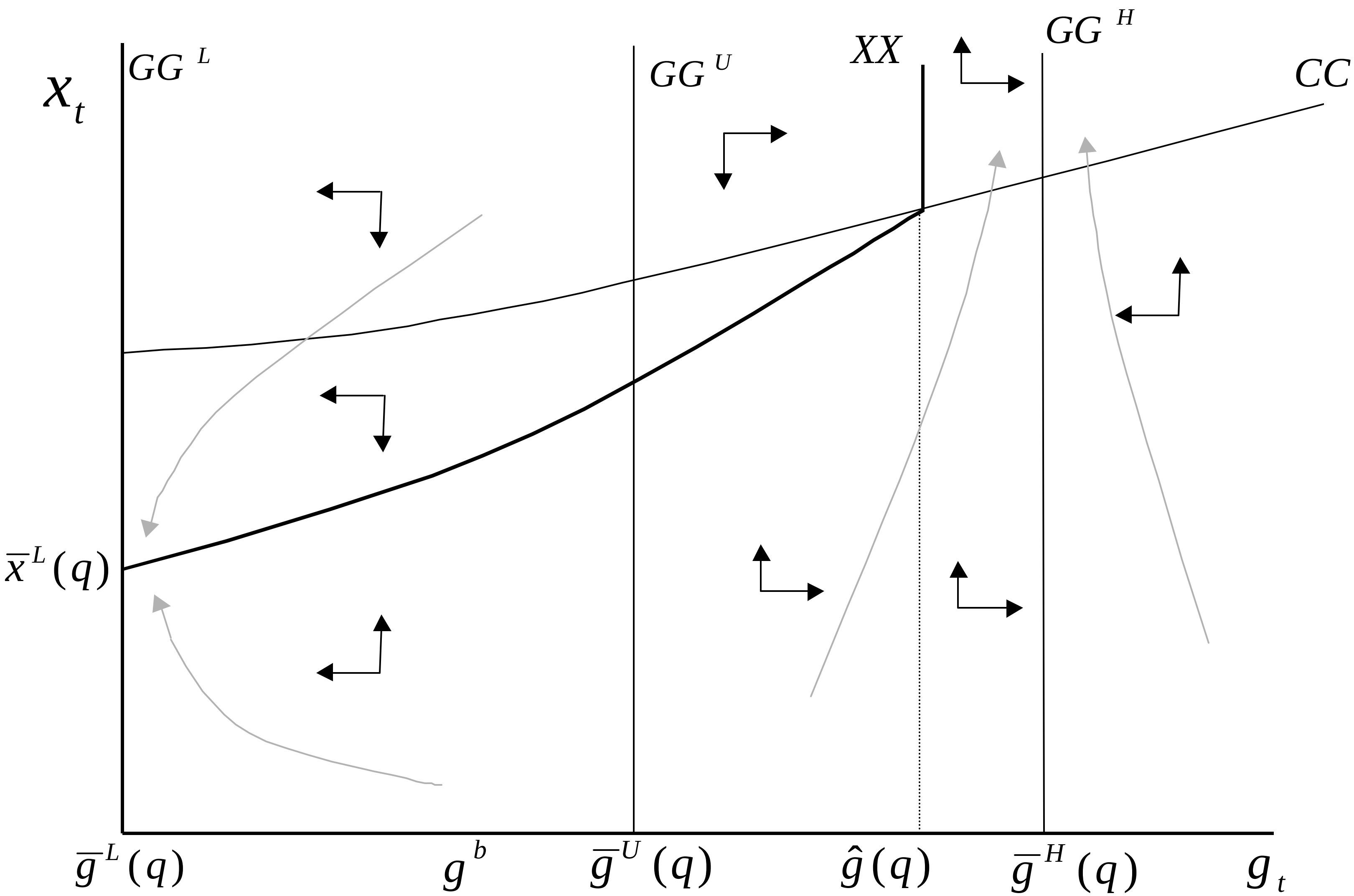

Figure 5(b). The Evolution of Technological Progress and Effective Resources

$$q \in (0, \hat{q})$$

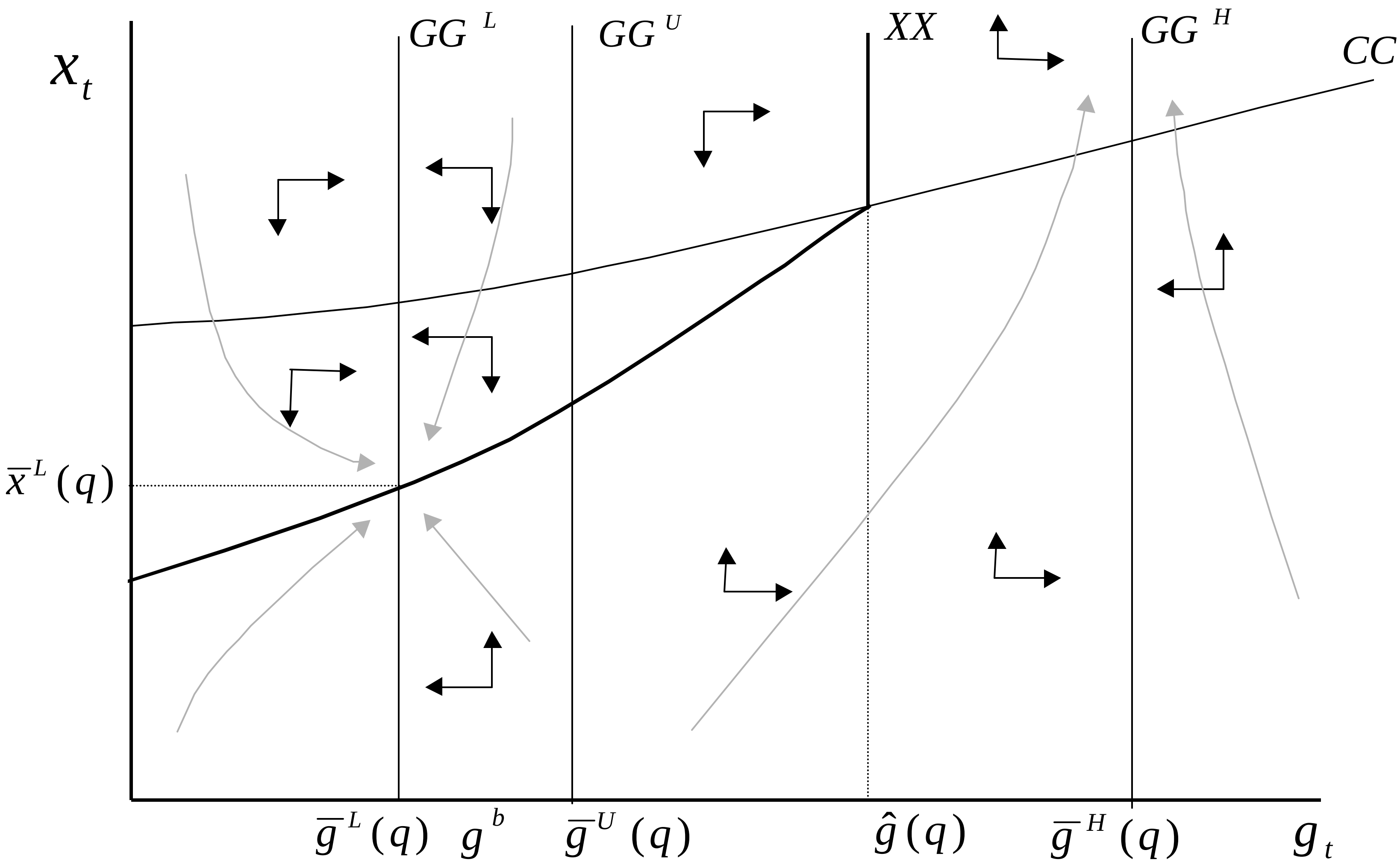

Figure 5(c). The Evolution of Technological Progress and Effective Resources

$q > \hat{q}$