Migration of Firms, Home Bias and the Geographical Distribution of Growth

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Foreign investment and the activity of foreign firms can have a substantial effect on an economy. This study examines the short run and long run effects of the movement of locally owned firms from a developed country to an undeveloped country on the output and growth rate of each in the presence of home bias. The paper analyzes the direction of movement of firms over time, firm ownership, GDP, GNP, wages and long run growth rates using a model in which the source of growth is the increase in the number of firms (which produce with decreasing marginal productivity). Various government policies towards the entry of foreign firms are examined and it is found that for the undeveloped country harsher policies towards entering firms lead to better results in the long run. Counter-policies are briefly discussed.

1 Introduction

Multinational firms are accountable for an increasingly large share of world trade. Foreign direct investment (FDI)\(^1\) is increasing more rapidly than income (especially in the developing countries) and is thus becoming a dominant factor in determining a country’s economic performance.\(^2\) This is occurring despite the home bias phenomenon in which preference is given to investment in one's home country over investment in foreign countries, even when this is not the optimal decision.\(^3\)

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1 An analysis of FDI and Foreign Portfolio Investment can be found in Goldstein & Razin (2005).


The movement of firms between countries, which is analogous to labor migration between countries,\(^4\) has a substantial effect on their economies. Countries react to the entrance of foreign firms in a variety of ways, ranging from the encouragement of entry to the discouragement of entry through various ways of taxation. Thus, while many countries encourage the entry of firms through subsidies and tax exemptions (Ireland, for example, used low tax rates for foreign firms), others, such as China and India, 'tax' foreign firms through requirements for partial local ownership or the sharing of knowledge.\(^5\)

Multinational firms have previously been analyzed in horizontal as well as vertical frameworks. The horizontal approach, in which firms locate production plants in several countries, was adopted in Markusen (1984, 1995), who analyzed national and multinational enterprises with firm-level knowledge. A similar approach appears in Markusen & Venables (1998). A different approach, which separates headquarters from production, appears in Helpman (1984). Grossman & Helpman (1991) explored an international economy with monopolistic competition and sustained endogenous growth in quality or variety of products, in which firms can separate activities across countries. Grossman, Helpman & Szeidl (2003) examined the firm's decision regarding the production and assembly of an intermediate good in a framework with different ex-post productivity for each firm. In a similar framework, Antràs & Helpman (2004) analyzed the firm's decision regarding integration vs. the outsourcing of intermediate goods production in a global economy\(^6\) and Grossman & Helpman (2005) studied the firm’s

\(^4\) Aharonovitz (2006) deals with the effects of the movement (migration and commuting) of workers.
\(^6\) A similar question concerning ownership and control is analyzed in Feenstra & Hanson (2005).
choice of outsourcing partner. Alvarez & Lucas (2004) analyzed gains from trade in an
economy with different levels of productivity.\footnote{See also Eaton & Kortum (2002).}

Though a great deal of research has been done on the multinational firm’s
decision making and on the effect of multinational firms and international trade on
economic growth, little has been said about the effect of ownership on the firms in this
framework, particularly in the presence of home bias. The aim of this study is to analyze
the short and long run effects of a (possible) movement of locally owned firms from a
developed country to an undeveloped country on their output and growth rate.

In this paper, a model is constructed in which the production of a single good
takes place in a given number of firms (which are using labor as the only production
factor), each with decreasing marginal productivity. Growth is the result of an increase in
the number of firms which depends positively on both profitability and the number of
firms (which represents the stock of knowledge relevant to establishing a new firm). In a
framework with one developed country that has many firms and one less developed
country that has no firms, firms may migrate from the former to the latter in order to
increase their profitability, despite the home bias of their owners. However, in later
periods, firms may migrate back from the (now developed) country to their former home.

Three policies towards the entry of firms are analyzed. The first, which allows
firms to freely enter, leads to the same growth rate in both countries in the long run,
though the less developed country always remains with a lower wage and level of output
than the developed country. The second, in which the undeveloped country requires
partial local ownership, leads to a higher growth rate in the short run in the less-
developed country and eventual convergence of the wage, GDP and growth rates (though
not of GNP) in the two countries. The last policy to be analyzed requires an increasingly larger share of local ownership over time. It results in a faster growth rate of the undeveloped country and eventual convergence of the wage, GDP, GNP and growth rates of the two countries, and may even lead to the undeveloped country overtaking\(^8\) the developing country in GNP.

The rest of the paper is organized as follows: Section 2 presents the model. Section 3 analyzes the equilibrium under various government policies. Section 4 concludes.

2 The Model

In order to analyze the effect of the migration of firms, a simple growth model is constructed. The model is first presented for the case of a single country in order to demonstrate its characteristics and is then expanded to the two-country case.

2.1 A Single Country Growth Model

Assume a single country with a labor force of size \(L\) which remains unchanged in every period. Output is produced by several firms which all manufacture the same product\(^9\) using labor as the sole input. The number of firms operating in each period is denoted by \(n_t\).

The production function, \(f(l)\), which is common to all firms, has positive but decreasing marginal productivity and satisfies Inada\(^{10}\) (1963)’s conditions. For simplicity, the following functional form is assumed:\(^{11}\)

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\(^8\) For an example of overtaking, see Brezis, Krugman & Tsiddon (1993).

\(^9\) Using a variety of products which are perfect substitutes and have the same production function (and therefore the same price) yields a similar result.

\(^{10}\) \(f'(0) = \infty, f'(\infty) = 0\)
Firms operate under perfect competition but there is no free entry of new firms. Since perfect competition is assumed and all firms are alike, each firm employs 

\[
\frac{L}{n_t} \text{ workers who are paid the value of their marginal productivity:}
\]

\[
w_t' = \alpha \left( \frac{L}{n_t} \right)^{\alpha - 1}
\]

Therefore the profit of a particular firm is:

\[
\pi_t(n_t) = f(l_t) - w_t l_t = (1 - \alpha) \left( \frac{L}{n_t} \right) \alpha
\]

Note that profit is decreasing in the total number of firms. Since firms are continuously opening new facilities while closing old ones, it is assumed that each firm exists for only one period and that at the end of the period its production facilities become obsolete. At the end of the period the "parent" firm is divided into several new firms that operate in the next period under the same ownership (possible changes in the ownership of the new firms are discussed in sub-Section 2.2). There is no (free) entry of other firms into the market since the knowledge and facilities necessary for production are obtainable only from the operation of an existing firm. The number of new firms created from an existing one depends on the profit of the firm and on the total number of firms. Thus, the more profitable a firm is, the more new firms it can establish and the more firms that exist, the easier (though less profitable) it is to establish new firms (since the number of firms represents the stock of knowledge).

\[\text{Similar results (though without a constant rate of growth) can be derived for a more general production function which satisfies the Inada conditions.}\]
available for use in the establishment of new firms out of existing firms). This last assumption is similar to the commonly made assumption that knowledge is a public good (see for example Grossman and Helpman (1991:57-62)). Denote the number of firms created as \( g(n, \pi(n)) \), where \( g(n, \pi) > 1 \) for positive \( n \) and \( \pi \), and \( g_{n} > 0, g_{\pi} > 0 \) though \( \pi \) is decreasing in \( n \). Moreover, it is assumed that the increase in \( g \) due to an increase in the number of existing firms is exactly the same as the decrease in \( g \) due to the decrease in profits (which is also a result of that same increase in the number of existing firms):

\[
\forall n, \quad g(n, \pi(n)) = g(n, (1-\alpha)\left(\frac{L}{n}\right)^{\alpha}) = k, \quad \text{where } k \text{ is a constant}.\]

The total GDP of the economy in period \( t \) is, therefore:

\[
Y_t = n_t f(l_t) = n_t^{1-\alpha} L^\alpha
\]

The total payment to labor is \( w_t L = \alpha L^\alpha n_t^{1-\alpha} \), which corresponds to a labor share of output equal to \( \alpha \). The corresponding share of firms is \((1-\alpha)\).

The consumers, who are also the workers and the owners of the firms, live for a single period \( (L \text{ consumers in each period}) \), without any possibility of saving. Therefore, their demand for consumption equals their income (GNP) and therefore the market clears. Note that Walras' Law allows for the analysis of the labor market's equilibrium without analyzing the market for goods.

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12 Note also that the increase in the number of firms assumed here resembles the constant increase in the number of products in their model.
13 Another way of modeling this would have been to assume that an existing production facility continues to operate in the next period and that \((g-1)\) new facilities are created from each existing firm. See Section 4 for further discussion of the differences between the two methods.
14 For example, \( g(n, \pi) = k \frac{n^{\alpha} \pi}{(1-\alpha)L^\alpha} \).
Proposition 1: The wage and GDP grow at the same constant rate.

Proof: The growth rate of output depends on the growth rate of the number of firms which is constant:

\[
    \frac{Y_{t+1}}{Y_t} - 1 = \frac{n_{t+1}^{1-\alpha} L^\alpha}{n_t^{1-\alpha} L^\alpha} - 1 = k^{1-\alpha} - 1
\]

A similar equation for the wage yields the identical growth rate.

QED

Moreover, since the number of firms in period \( t \) is \( n_t k^{1-1} \), one can easily find the wage, profit and output for each period. Note that the total profit of all firms grows at the same rate, but since the number of firms grows even faster, the profit of a single firm decreases over time.\(^{15}\)

So far I have analyzed the model for the simple case of a single country. I turn now to analyze the case of two countries and examine the mutual influences between them.

2.2 The Complete Model

Assume a world with two countries, \( H \) and \( F \), each with a labor force of size \( L \). The output in each country is produced, as before, by several firms manufacturing the same product using labor as the sole input. The number of firms in each period in each country is denoted by \( n_i^t \), \( i=H,F \). The production function is the same as before:

Each firm employs \( \frac{L}{n_i^t} \) workers who are paid their marginal productivity:

\(^{15}\) As before, this result resembles the case of product variety; see, for example, Grossman & Helpman (1991:57-62).
Therefore the profit of a particular firm is:

\[ \pi_i(n_i') = (1 - \alpha) \left( \frac{L}{n_i'} \right)^\alpha \]

Each firm exists for only one period and at the end of the period it is divided into \(g\) new firms, where \(g\) depends on the profit of the firm and on the total number of firms in the world. Thus, a more profitable firm grows faster and therefore has a faster multiplication rate. The more firms there are, the more knowledge is available for the establishment of new firms and therefore the easier it is to do so. Since a free flow of knowledge is assumed between the two countries, the multiplication rate is affected by the total number of firms in the world.\(^{16}\) Therefore, one obtains \( g(n, \pi(n')) \) with the same assumptions as before. A necessary additional assumption (though only a technical one) is that the multiplication rate is not excessively high:

\[ \forall n_i > n_i', n_i' g(n_i' + n_i', \pi(n_i')) > n_i g(n_i' + n_i', \pi(n_i')) \]

This assumption ensures that when the number of firms in one country is fewer than that in the other, it will not exceed it in the next period but only move towards it, since otherwise repeated overtaking might occur. A continuous model would not require such an assumption. The example from the previous section is relevant here as well.

At the beginning of each period, the newly established firms choose where (i.e. in which country) to locate their activity. This decision is made by the owners of the firms

\(^{16}\) This assumption is similar to that of international spillovers of knowledge; see for example Grossman & Helpman (1991:178).
and is affected by a home bias, where the ownership of a firm can be only \( H \) (citizens of the home country), only \( F \) (citizens of the foreign country) or any mixture of the two. In the case of mixed ownership, the share of ownership of investors from one country in the parent firm of a firm operating in period \( t \) will be denoted by \( B_i \), and therefore the share of ownership of investors from the other country will be \((1 - B_i)\). When no changes in ownership occur, these shares also represent the ownership situation of the firm which is actually operating in period \( t \). The home bias effect causes owners to demand higher profits in order to locate the activity of a firm outside of their own country. Therefore, the condition for locating a firm owned by investors in country \( i \) in country \( j \) is:

\[
\pi_i \geq \gamma \pi_i
\]

where \( \gamma > 1 \) stands for the preference of the owners for an activity in their own country, and for simplicity a symmetric preference between the two countries is assumed (i.e. \( \gamma \) is equal for both countries). In a similar manner, when the share \( B_i \) of a parent firm is owned by investors from country \( j \) (and the rest by owners from country \( i \)), the condition for locating a firm just created from it in country \( j \) will be:

\[
(1 - B_i) \pi_i + B_j \pi_i \geq (1 - B_i) \gamma \pi_i + B_j \pi_i
\]

since each owner has a bias towards his home country.

Notice that the owners' decision is based solely on profits in the next period. However, sufficiently high discount rates can yield the same results (see Section 4 for discussion).

\[17\] See the Introduction for references and some elaboration.
Ownership of the new firms remains the same as that of the parent firm, unless the owners are willing to give up some of their ownership (see sub-Sections 3.2 and 3.3). In an identical situation to the one presented above, when owners are required to give up a share $\beta$ in order to operate in country $j$ (while no such demand exists in country $i$), the condition becomes:

$$
(12) \quad (1 - \beta)[(1 - B_i)\pi_i^t + B_j\gamma\pi_n^t] \geq (1 - B_i)\gamma\pi_i^t + B_j\pi_i^t
$$

where $B_i$ is the ownership of investors from country $j$ in the parent firm, i.e. the ownership prior to giving up $\beta$. The right side of (12) is the investors' profit from locating in country $i$ (adjusted for home bias) while the left side is the profit from locating in country $j$ adjusted for home bias and the requirement to give up a share of the firm.\(^{19}\)

During the evolvement of the economy two types of ownership can be created - direct or chained through other firms. However, we only need to keep track of the origin of the owners and not the form of ownership. Consider, for example, a firm in the first period owned by $H$ investors. Now assume that in the second period it multiplies into two companies, both of which operate in $H$. These two new companies can either be held directly by investors in $H$ (in this case there are two separate companies) or held by the first period “parent” company, which serves as a holding company of the two new companies (it has no other activity) and is itself owned by the investors in $H$ (thus, resulting in a multi-plant firm). In a similar manner, if one of the two new companies operates in $F$, then under the former form of (direct) ownership this is simply an

\(^{18}\) Another possible interpretation is that the new firms are established by senior managers of the parent firm, who acquired the necessary skills and knowledge, and the origins of those senior managers are divided between the countries in the same ratio as the owners. Therefore the new ownership is, on average, divided between the two countries in the same ratio as the previous one, although the owners themselves are different.

\(^{19}\) Notice that the condition refers only to the profit of the current period, see sub-Sections 3.2 and 3.3 and Discussion for an elaboration and for the case of the discounted stream of profits.
investment abroad, while under the latter form of ownership the first period firm becomes a multinational firm, with a subsidiary in $H$ and a subsidiary in $F$. The division of ownership between the countries affects the decision of where to locate as described above, but the method of holding (i.e. directly or through another firm) does not, and as a result we do not need to keep track of the form of ownership. Therefore, although both multi-plant and multinational firms evolve in this model as part of the growth process and the migration of firms between countries, it does not need to be analyzed.

The specification of consumption behavior will complete the model. The consumers in each country, who are also the workers and the owners of the firms, live for a single period ($L$ consumers in each period), without the possibility of saving. Therefore, total demand equals their total income (i.e. GNP) and the market will clear.

The next section considers the situation when an undeveloped country decides to allow the entry of foreign firms.

3 Entry Policies

Migration of firms has a significant effect on the economy, and therefore it is accompanied by government policy. When a country decides to allow foreign firms to operate within it, there are several (active trade) policies it can adopt, each leading to a different result. One option is to encourage entry through subsidization of the foreign firms, i.e. reduced tax rates and grants. At the other extreme is the policy of taxing foreign firms which can take the form of demanding the sharing of knowledge or requiring a share of the ownership for local citizens (possibly one that increases over time) even when local owners will not be contributing to the activity of the firm. The
effects of allowing entry are analyzed under several possible policy regimes when one 
country is developed and the other is undeveloped and the latter decides to allow foreign 
firms to operate within it.

Assume two countries, $H$ and $F$. In period $T$, prior to the possibility of entry, $n_r$ 
firms, owned by $H$ owners, operate in $H$. There are no firms owned by residents of $F$ and 
therefore no firms operate in $F$ prior to the possibility of entry. At the beginning of period 
$T$, country $F$ decides to allow foreign firms to operate within it, either with or without 
restrictions.

### 3.1 Free Entry

One extent of policy is to encourage the entry of foreign firms through tax 
exemptions, grants, restrictions on the import of similar products, etc. The policy 
analyzed here is a more 'neutral' one, where country $F$ simply allows foreign firms to 
operate within it without restriction. Since all the firms are owned by residents of $H$, the 
entry condition in (10) applies and firms will move to $F$ until

$$(1 - \alpha) \left( \frac{L}{n_F} \right)^\alpha = \gamma (1 - \alpha) \left( \frac{L}{n_H} \right)^\alpha, \text{ or:}$$

$$(13) \quad n_H = \frac{\gamma^{1/\alpha} n_r}{1 + \gamma^{1/\alpha} n_r}, \quad n_F = \frac{1}{1 + \gamma^{1/\alpha} n_T}. \quad \text{Proposition 2: A free entry policy immediately improves the wage, GDP and GNP in } F \quad \text{and leads to continuing growth in those variables but } F \text{ does not catch up to } H \text{ in any of these variables.}$$

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\[20\text{ See UNCTAD (2002: 204-208) for a description of the various incentives and for detailed description of policy in Ireland and Malaysia.}\]
**Proof:** According to (13), the number of firms in $F$ grows from zero to $n_{F}^{T}$. As a result, the wage, GDP and GNP grow to
\[ \alpha \left( \frac{L}{n_{F}} \right)^{\alpha-1}, \] (\(n_{F}^{T}\))^{\alpha} L^{\alpha} and \[ \alpha(n_{F}^{T})^{1-\alpha} L^{\alpha}, \] respectively. Notice that since $n_{F}^{H} > n_{F}^{T}$ (since $\gamma > 1$) and the companies located in $F$ are owned by residents of $H$, the wage, GDP and GNP will be higher in $H$.

Since firms can freely migrate in and out of $F$ and ownership remains unchanged, equation (13) holds for every period $t$, $t \geq T$ and the same ratio of $n^{H}$ to $n^{F}$ will prevail in every period. The long run growth rate of the wage, GDP and GNP is
\[ \left( \frac{n_{F_{t+1}}}{n_{F_{t}}} \right)^{1-\alpha} - 1 \]
which is equivalent to
\[ \left( \frac{n_{t+1}}{n_{t}} \right)^{1-\alpha} - 1 \]. Therefore, the wage, GDP and GNP grow at the same rate in both countries and thus remain higher in country $H$.

QED

Note that since in every $t \geq T$, $g(n_{i}, \pi(n_{i}^{F})) > g(n_{i}, \pi(n_{i}^{H}))$, 21 firms in $F$ multiply faster than those in $H$. However, since the proportion of firms operating in $F$ (out of the total $n$) remains the same, firms are continually migrating back to $H$. Note also that since the demand for consumption equals GNP (see sub-Section 2.2), the goods market in each country is cleared with the import (or export) of goods equal to the difference between GNP and GDP.

The short run effects on $F$ are straightforward. The wage, GDP and GNP of country $F$ rise and can rise even further if a policy of subsidization is implemented in that period. In the long run, country $F$’s rate of growth depends on the growth in the number

\[ n_{i}^{F} < n_{i}^{H}: \] therefore $\pi(n_{i}^{F}) > \pi(n_{i}^{H})$ and $g(n_{i}, \pi(n_{i}^{F})) > g(n_{i}, \pi(n_{i}^{H})).$
of firms, which is between $g(n_t, \pi(n^F_t))$ and $g(n_t, \pi(n^H_t))$, but $F$ will always lag behind $H$ in all three variables. \footnote{It should be mentioned that a single period subsidy to foreign firms followed by a free entry policy yields similar long run results.} Figure 1, in which entry is permitted from period 6 onward, demonstrates this. Thus, in period 6 there is an improvement in $F$’s GDP and GNP and continuous growth thereafter (at the same rate in both countries). Note that $F$’s variables are always lower than those of country $H$.

**Figure 1 Here**

### 3.2 Joint Ownership

Many countries demand a share of local ownership as a prerequisite for the entrance of a foreign firm. This is done, among other reasons, in order to encourage local industry, to better monitor the operations of foreign firms or can simply be the result of corruption. \footnote{See Wei & Shleifer (2000: 311-313).} Therefore, the second policy option to be analyzed is the requirement of a one-time tax in the form of joint ownership in exchange for the right to operate in $F$. When a fully $H$-owned firm wishes to operate in $F$, it must give up a share $\beta$ of its ownership to local owners. Firms established from that firm in the following periods can operate in $F$ (without further "taxation") or migrate out of $F$ while retaining the same division of ownership. They can, of course, freely return to $F$ later since $\beta$ of their ownership is held by residents of $F$. 
Under this policy regime, when country F begins to allow entry, the condition that induces firms to enter is similar to (12) except that \( B=0 \) since all the firms are owned by residents of H:\(^{24}\)

\[
(1 - \beta) \pi_T^F \geq \gamma \pi_T^H
\]

and firms will continue moving to F until
\[
(1 - \beta)(1 - \alpha) \left( \frac{L}{n_T^F} \right)^\alpha = \gamma (1 - \alpha) \left( \frac{L}{n_T^H} \right)^\alpha
\]

or:

\[
n_T^H = \left( \frac{\gamma}{1 - \beta} \right)^{\frac{\alpha}{1 - \beta}} n_T^F
\]

\[
n_T^F = \frac{1}{1 + \left( \frac{\gamma}{1 - \beta} \right)^{\frac{\alpha}{1 - \beta}}} n_T^F
\]

**Proposition 3:** A policy of joint ownership leads to an immediate improvement in F’s wage, GDP and GNP and continuing growth in those variables in subsequent periods. Catch-up in wage and GDP is possible for a sufficiently large \( \beta \) (i.e. share of local ownership) but there can be no catch-up in GNP.

**Proof:** As in Proposition 2, the wage, GDP and GNP grow immediately in period \( T \) to

\[
\alpha \left( \frac{L}{n_T^F} \right)^{\frac{\alpha - 1}{\alpha}}, \quad L^F (n_T^F)^{\frac{1 - \alpha}{\alpha}}, \quad \text{and} \quad \alpha L^F (n_T^F)^{\frac{1 - \alpha}{\alpha}} + \beta (1 - \alpha) L^F (n_T^F)^{\frac{1 - \alpha}{\alpha}},
\]

respectively. Notice, however, that under this policy regime the proportion \( n_T^F \) of \( n_T \) is smaller than under the previous policy.

Since \( n_T^H > n_T^F \) (because \( \gamma > 1 \) and \( \beta < 1 \)), \( g(n_T, \pi(n_T^F)) > g(n_T, \pi(n_T^H)) \), firms in F multiply faster than those in H and profits decrease at a faster rate. Furthermore, this situation cannot be reversed to \( n_T^H < n_T^F \), \( t > T \) as can be seen from equation (9). As long

\(^{24}\) If firms discount future profits, the demand for local ownership also implies loss of some future profits, as well as loss of control. Therefore, firms request higher compensation (i.e. higher current profits) in order to migrate to F, resulting in lower (yet positive) number of migrating firms (and higher profits in F).
as this situation prevails, the entrance condition (14) will not hold for fully H-owned firms and new firms will not enter country F. However, since firms operating in country F are now partially locally owned, migration out of F needs to be carefully analyzed.

The condition for migrating back to H for firms with a share $\beta$ of local ownership is:

\[(16) \quad (1 - \beta)\pi_{t+1}^F + \beta\gamma\pi_{t+1}^F < (1 - \beta)\gamma\pi_{t+1}^H + \beta\pi_{t+1}^H + \beta, \quad \text{where } t \geq T\]

which directly yields:

\[(17) \quad \pi_{t+1}^F < \frac{(\gamma + \beta - \gamma\beta)}{(1 + \gamma\beta - \beta)}\pi_{t+1}^H\]

If $\beta \geq \frac{1}{2}$, firms require that $\pi_{t+1}^F < \pi_{t+1}^H$ in order to migrate to H. However, since initially $n_t^H > n_t^F$ and this situation cannot be reversed (since as $n^F$ approaches $n^H$ its growth rate decreases and once they equalize so do the two growth rates, while equation (9) ensures that there can be no reversal between the two), the reversal of profitability cannot occur and there is no migration to H. The share of $n^F$ in $n$ approaches one half.

To see this, define $\varepsilon, \varepsilon > 0$, as a desirable difference between $n^H$ and $n^F$ ($\varepsilon$ can be chosen as arbitrarily small), and note with $\Delta, \Delta > \varepsilon$, the current (actual) difference between $n^H$ and $n^F$. Every $\varepsilon$ produces a difference in the multiplication rates of the firms in the two countries which, for high enough $t_0$, reduces $\Delta$ to $\varepsilon$ within $t_0$ periods. Since the actual difference in the multiplication rates is higher (since the actual difference in the number of firms is greater than $\varepsilon$), the number of periods required to achieve $\varepsilon$ is smaller than $t_0$. The difference between $n^H$ and $n^F$ approaches 0 and therefore the share
of \( n^F \) in \( n \) approaches one half. As a result, the wage and GDP equalize between the two countries while the GNP of \( H \) remains higher due to the ownership of firms operating in \( F \). The long run growth rate of \( n \) approaches \( g(n, \pi(0.5n)) \).

If \( \beta < \frac{1}{2} \), equation (17) implies that firms begin migrating to \( H \) even when \( \pi^F_{t+1} < \pi^H_{t+1} \). Since the share of \( F \) firms is less than half, their multiplication rate is higher than that of \( H \) firms and the gap in profits becomes increasingly smaller (as above). Once \( \pi^F \) is close enough to \( \pi^H \), firms start to migrate back to \( H \), but this time with a share of \( F \) ownership. The share of firms in \( F \) will remain stable from that time on, but since they multiply faster, out-migration will persist. Note that although country \( F \)'s wage, GDP and GNP grow continuously, they will remain smaller than those of country \( H \).\(^{25,26}\)

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In the short run, the wage, GDP and GNP increase. Note that the wage and GDP are smaller under this policy than under the free entry policy, whereas GNP may be either higher or lower, depending on the size of \( \beta \).

In contrast, long run results are more positive than under free entry. Since fewer firms migrate to \( F \) initially, their multiplication rate is higher and thus so is the economy's growth rate, although it decreases over time. For \( \beta \geq \frac{1}{2} \), \( F \)'s growth rate is higher than that of \( H \), but decreasing over time so that the share of firms operating in \( F \) approaches one half and the wage and GDP of the two countries equalize over time. The

\(^{25}\) Note again that according to the assumptions made with regard to consumers, the market for goods is cleared in every period.

\(^{26}\) If firms discount future profits, smaller (but positive) number of firms migrate to \( F \) in the first period, resulting in lower wage and GDP than those computed in the proof for that period (but still, higher than without entrance). The rest of the proof and the result with regard to the long-run remain unchanged.
GNP of $F$ remains lower, since the firms there are partially owned by $H$ (see figure 2).

For $\beta < \frac{1}{2}$, the multiplication rate of $F$ firms remains higher, but some $F$ firms (with the proportion $\beta$ of $F$ ownership) migrate out from a certain point in time onward, so that the wage, GDP and GNP in $F$ remain lower. However, a comparison of equations (17) and (10) shows that a larger share of firms operate in $F$ under this policy (since $\frac{\gamma + \beta - \gamma \beta}{1 + \gamma \beta - \beta} < \gamma$) and all three variables are closer to those of country $H$ relative to the previous policy of free entry.

3.3 Increase in Local Ownership over Time

Firms operating in a foreign country may be subject to an ongoing increase in the share of local ownership or an increase in the local activity in the same sector. This can be the result of, among other things, a government policy to encourage local workers to open their own firms or to encourage spillovers of knowledge to local firms (by not protecting intellectual property) or may simply be the result of ongoing corruption. Whatever the reason, the result is an ongoing increase in local ownership, which is equivalent to a periodical 'tax' on the original owners. Therefore, the final policy to be analyzed is the requirement of an increasing share of local ownership each period in exchange for the right to operate in $F$. When a firm chooses to operate in $F$ in a particular period, the previous owners (the owners of the "parent" firm) are left with only $1 - \beta$ of the new firm, while a share of $\beta$ is allocated to local owners. This is repeated in every
period in which the firm operates in $F$, whether its "parent" firm operated in $H$ or $F$, and regardless of its share of $F$ ownership. I assume that $\beta$ is not too large:

$$\beta \leq 1 - \frac{1}{\gamma}$$

Under this policy regime, since all firms are of $H$ ownership, when $F$ begins allowing entry and since firms maximize the profit of only the current period\textsuperscript{27}, the entrance condition remains the same as in the previous case (i.e. equation (14)) and firms move to $F$ until

$$(1 - \beta)(1 - \alpha) \left( \frac{L}{n_T} \right)^\alpha = \gamma(1 - \alpha) \left( \frac{H}{n_T} \right)^\alpha,$$

and $n_T^H$ and $n_T^F$ equal their values in equation (15).

**Proposition 4:** A policy of increasing local ownership over time leads to an immediate improvement in $F$'s wage, GDP and GNP. $F$ will catch up in these variables in the long run. Whether it overtakes $H$ depends on $g$.

**Proof:** As in Proposition 2, the wage, GDP and GNP grow immediately in period $T$ to

$$\alpha \left( \frac{L}{n_T} \right)^{\alpha-1}, L^\alpha (n_T^F)^{1-\alpha}, \text{ and } \alpha L^\alpha (n_T^F)^{1-\alpha} + \beta (1 - \alpha) L^\alpha (n_T^F)^{1-\alpha},$$

respectively. Notice that all three variables are of the same magnitude as in the case of the previous policy (i.e. constant local ownership). Since $n_T^H > n_T^F$ (because $\gamma > 1$ and $\beta < 1$),

$$g(n_T, \pi(n_T^F)) > g(n_T, \pi(n_T^H)),$$

firms multiply faster in $F$ than in $H$ and profits decrease faster in $F$ than in $H$. As long as this situation continues, the entry condition in (14) will not hold for any fully $H$-owned firms and new firms will not enter $F$.

\textsuperscript{27} See Section 4 for a discussion of the discounted stream of profits case.
Firms that choose to migrate from $F$ back to $H$ will not return to $F$. Firms that migrate out of $F$ with local ownership of $\overline{B}$ (the ownership of their parent firms which equals the ownership of the new firms if they leave $F$) satisfy:

$$ (1 - \beta)[(1 - \overline{B})\pi^F + \overline{B}\gamma\pi^F] < (1 - \overline{B})\gamma\pi^H + \overline{B}\pi^H, $$

which can be rearranged as:

$$ \frac{\pi^F}{\pi^H} \leq \frac{\gamma + \overline{B} - \overline{B}\gamma}{(1 - \beta)(1 - \overline{B} + \overline{B}\gamma)}, $$

and migration back to $H$ occurs until (20) is equalized. Note that:

$$ \frac{\pi^F}{\pi^H} = \left( \frac{n^H}{n^F} \right)^\alpha $$

Since the share of local $F$ ownership among firms that stayed in $F$ increases over time, the right side of (20) decreases (for any $B > \overline{B}$) and for the same profit ratio there will not be any additional migration back to $H$, which would have increased the left side of (20). The multiplication rate of firms in $F$ is higher than in $H$ and therefore without such migration the ratio of the firm proportions (right side of (21)) decreases and therefore so does the left side of (21) and (20). Therefore, the exit condition expressed in equation (19) for firms with $F$ ownership of $\overline{B}$ continues to be valid and there is no return migration to $F$ of those firms.

Notice that the share of $F$ ownership in firms that decide to continue operating in country $F$ increases over time and eventually approaches 1. Since there are no firms entering $F$ in the periods following $T$, the share of $F$ ownership in all firms that operate in $F$ approaches 1.

In the long run, since local $F$ ownership approaches 1, the exit condition becomes:
but since \((1-\beta)\gamma>1\), this last condition requires that \(\pi^F<\pi^H\), or \(n^F>n^H\), a condition that contradicts assumption (9). Therefore, there is no migration from \(F\) back to \(H\) in the long run and the share of firms in each of the two countries approaches one half (as in Proposition 3 with \(\beta\geq\frac{1}{2}\)) and therefore the wage and GDP in each of the two countries approach equality.

Overtaking in GNP requires that some firms (with a share of \(F\) ownership) migrate out of \(F\). The condition for this to occur is given in equation (12), which can be rearranged as:

\[
\frac{\pi_{t+1}^F}{\pi_{t+1}^H}<\frac{(\gamma+B_t-\gamma B_t)}{(1-\beta)(1-B_t+\gamma B_t)} \quad \text{(see also equation (20))}
\]

According to (21), for some \(g\) functions the left side of equation (23) is close enough to 1 (when \(B\) is small enough), such that migration from \(F\) back to \(H\) can occur. For example, if the left side of (23) is close enough to 1 for \(t=T\), (23) becomes

\[
1<\frac{\gamma+B_t-\gamma B_t}{(1-\beta)(1-\beta+\gamma B_t)}
\]

which is valid for every \(\beta\) as long as \(1<\gamma<1.5\). Therefore, for some values of \(\gamma\) and functional forms of \(g\), migration from \(F\) back to \(H\) is possible. In such cases, overtaking in GNP occurs since in later periods the number of firms operating in each country equalizes and some firms which operate in \(H\) are (partially) \(F\) owned. Otherwise, the GNPs of the two countries approach equality, as do the wages and GDPs.
Notice that if the left side of (23) does not approach 1 rapidly enough, then as $B$ approaches 1 the right side of (23) equals \( \frac{1}{(1 - \beta)\gamma} \) which is smaller than 1 (see equation (18)) and there is no return migration or overtaking.  

QED

Figure 3 describes the development of GDP and GNP in both countries under this policy with entry beginning in period 6. Notice that while the GDPs of the two countries approach equality (as does the number of firms operating in each country), the GNP of $F$ overtakes that of $H$. This occurs because, as a result of the specific $g$ function used here, several firms with a share of $F$ ownership migrated back to $H$ and the ownership in $F$ approaches pure local ownership over time. Note that though the overtaking in GNP seems negligible in the graph, the logarithmic scale is misleading and the difference in that example is in fact about 12%.

**Figure 3 Here**

**Proposition 5:** Overtaking in GNP can be achieved for every $g$ satisfying the previous assumptions using a more complicated policy scheme.

**Proof:** Assume a policy scheme similar to the one analyzed in which a requirement for increasing local ownership $\beta$ over time that satisfies (18) is enforced in every period except period $S$, $S>T$, where $S$ is chosen so that a firm's profit in each of the countries is similar in that period. Denote the requirement for local ownership in $S$ as $\beta_1$, such that:

---

Note again that, as in the previous sub-section, the market for goods is cleared in every period.
and therefore $1 > (1 - \beta_i)\gamma$. The condition for migration from $F$ to $H$ at the beginning of period $S$ is:

$$1 - \beta_i) (1 - B + B\gamma) \pi^F_S < ((1 - B)\gamma + B) \pi^H_S,$$ as compared to (12).

Since without the change in $\beta$, $\pi^F_S \rightarrow \pi^H_S$ and $B \rightarrow 1$, equation (25) approaches $(1 - \beta_i)\gamma < 1$ when $S$ is sufficiently high. Therefore, since this condition is satisfied by (24), one can choose $S$ high enough so that (25) is satisfied and some firms with $F$ ownership will migrate to $H$.

In the next period, the requirement for additional $F$ ownership is reduced again to $\beta$. Since firms in $F$ multiplied faster in period $S$, not all the firms born from the above migrant firms would return to $F$ so that some of the firms will stay in $H$. As in Proposition 4, the number of firms in each country approaches equality, as do the wage and GDP. However, $F$’s GNP is now higher since some of the firms operating in $H$ are partly $F$-owned, while the ownership of the firms operating in $F$ approaches pure local ownership.

QED

In the comparison of this policy to the previous one (i.e. a fixed share of local ownership), short run (first-period) results are identical (for the same $\beta$), while long run results may differ. In the intermediate run, if $g$ is such that the gap between the two countries is closing rapidly, then under the increasing local ownership policy, there is return migration of firms and therefore medium range wage and GDP may be lower. However, long run results are more favorable, though they may take longer to achieve
due to the aforementioned return migration, since they include catch-up in all parameters and possible overtaking in GNP. Note, however, that the comparison in the short run and in the intermediate run (convergence path) is problematic, because the optimal $\beta$ differs between the policies ($\beta \geq \frac{1}{2}$ is optimal in the former and $\beta \leq 1 - \frac{1}{\gamma}$ in the latter).

4 Discussion

In the model that was presented, growth is generated by an increase in the number of firms which depends on both the profitability of firms and the stock of knowledge, as represented by the number of firms. In a two-country framework, where one country is developed and has many firms and the other is undeveloped and has no firms, it was shown that the migration of profit-maximizing firms from the developed to the undeveloped country (despite the owners' home bias) exported the growth process to the undeveloped country, thus putting the developed country at risk of losing its advantage. Three entry policies were analyzed for the undeveloped country with the result that a harsher policy yields better long-run results. While all policies improve the immediate term situation29 and yield equal long run growth rates for the two countries, the free entry policy leaves the undeveloped country with a permanent lag behind the developed country. The one-time tax policy (i.e. joint ownership) leads to convergence in GDP and a harsher policy of ongoing taxation (in the form of increasing local ownership over time) results in convergence in GDP and overtaking in GNP. Therefore, according to this model a policy of taxation is recommended as opposed to the policy of subsidization adopted by many countries.

29 Note, however, that the first policy analyzed yielded more favorable short run results.
Notice, however, that this recommendation holds in the case in which foreign firms, once they have begun operating in the undeveloped country, can produce and grow. An appropriate education level among the population, protection of property rights, a developed infrastructure and other similar conditions for this process to occur were assumed to exist. Otherwise, the growth process does not occur.

The aim of the policy (i.e. catching up with or overtaking the other country) and the policy recommendation with regard to the long run get an interesting interpretation when the situation is different than two equally-sized countries. If the developed world is large relative to a small undeveloped country, the latter does not affect the world output and growth rate. In that case, overtaking also means achieving the highest output in the long run. If there are several undeveloped countries, those countries might compete on migrating firms by offering various benefits. Countries that will not offer benefits but rather take a harsher policy would get fewer entering firms (but still, a positive number, since the profits of a single firm increase when the number of firms decreases), and therefore lower short-run output and wage, but they enjoy better long run result. Since some firms do enter, the long run result of the harsher policy (increase in local ownership over time) would still be overtaking.

Two simplifying assumptions need to be discussed here. The first is that during the growth process a firm multiplies into $g>1$ firms and that all new firms are able to migrate. An alternative to this assumption would have been that the firm continues to exist and that a positive number of new firms are created (with only new firms being able to migrate). Results in this case would be similar, but perhaps achieved at a slower pace. Thus, when entry becomes possible, migration to the undeveloped country could continue
for more than one period since the share of migrating firms that equalizes profits may be larger than the share of new (migration-able) firms. However, long run results remain unchanged.

Another simplifying assumption of the model is that firms base their decisions on profits only in the current period rather than discounting the entire stream of future profits. While under the first policy discounting future profits changes nothing and under the second one there is only a minor change in the first period (see sub-Section 3.2), under the last policy analyzed, in which firms that migrate to $F$ lose additional ownership in every period, accounting for future profits could affect the return-migration decision and therefore the model's result. Note, however, that since the relevant period in the model is long, an assumption of a high enough discount rate is a natural one. In such a case, when entry becomes permissible, the profit required to induce migration is higher and therefore less firms migrate. However, the process that follows remains the same as long as the home bias and the discount rate are sufficiently high relative to the periodic 'tax' (which is determined by the undeveloped country itself). Some firms will operate in $F$, and those firms would become locally owned. Since the difference in profits in the long run (as viewed by the firms in each country, i.e. adjusted for the home bias) between the current country a firm operates within and the other country is strictly positive, those firms would not migrate out and catching-up (and overtaking) will occur. Therefore discounting the stream of future profits does not affect the result with regard to the long run.

Finally, we turn briefly to the discussion of counter-policies that may be adopted by the developed country. The movement of firms from the developed country $H$ to the
undeveloped country $F$ leads to $F$ becoming the more developed country if it adopts the appropriate policy and $H$ allows the free entry of firms. This is a result of the behavior of profit-maximizing firms that does not take into account the firm’s effect on the long run balance between the two economies. There are a number of counter-policies that $H$ could adopt including, for example, the reduction of out-migration once entry is allowed either through direct restrictions on local firms, through subsidization or using a tax policy which increases $H$'s local ownership. Notice, however, that the former two policies merely postpones the overtaking while the latter leads to convergence instead of overtaking and does not enable the developed country to retain any lead that it once had. The issue of counter-policies and the reaction to them requires additional research, since it appears that liberal policies may not be recommended if a country wishes to maintain its economic leadership.
References


Figure 1 – GDP and GNP of $H$ and $F$ under Free Entry

Plotted for $L = 1$. $g(n, \pi) = 2 \frac{n^\alpha \pi}{(1 - \alpha)}$, $\alpha = 0.5$, $n^H_0 = 1$, $n^F_0 = 0$, $\gamma = 1.5$.
Figure 2 – GDP and GNP of $H$ and $F$ under Joint Ownership

Plotted for $L = 1$, $g(n, \pi) = 2 \frac{n^\alpha \pi}{(1 - \alpha)}$, $\alpha = 0.5$, $n_0^H = 1$, $n_0^F = 0$, $\gamma = 1.5$, $\beta = 0.6$
Figure 3 – GDP and GNP of \(H\) and \(F\) under Increasing Local Ownership

Plotted for \(L = 1\), 
\[
g(n, \pi) = 2 \frac{n^\alpha \pi}{1 - \alpha}, \quad \alpha = 0.5, \quad n_0^H = 1, \quad n_0^F = 1, \quad \gamma = 1.5, \quad \beta = 0.3
\]