

Cheap Children and the Persistence of Poverty*

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November 8, 2001

Abstract

This paper develops a theory of fertility and child educational choice that offers an explanation for the persistence of poverty within and across countries. The joint determination of the quality (education) and quantity of children in the household is studied under the key assumption that individuals' productivity as teachers increases with their own human capital. As a result, the poor choose high fertility rates with low education investment and therefore, their offspring are poor as well. Furthermore, the high fertility rates in poor economies dilute physical capital accumulation and amplify the effect of child quality choice on economic growth. The model generates multiple steady states even though the technologies employed in the production of human capital and output are convex and preferences are convex and homothetic.

Keywords: Fertility, Human Capital, Growth.

JEL Classification Numbers: J13, O11, O15, O40.

*I wish to thank Daron Acemoglu, Josh Angrist, Ilhem Baghdadli, Abhijit Banerjee, Patricia Crifo-Tillet, Oded Galor, Eric Gould, Yishay Maoz, Joram Mayshar, Olivier Morand, Avi Simhon, David Weil, Mathias Thoenig, Joseph Zeira, two anonymous referees, and participants in seminars in the Hebrew University of Jerusalem, MIT, and CESifo conference on Economic Growth and Inequality, Munich 2001, for helpful discussions. I am grateful to the Maurice Falk Institute for financial support.

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1 Introduction

This paper develops a theory of fertility and child educational choice that offers an explanation for the persistence of poverty within and across countries. The joint determination of the quality (education) and quantity of children in the household is studied under the key assumption that individuals' productivity as teachers increases with their own human capital. In contrast, the minimum time cost associated with raising a child regardless of the child's quality - the quantity cost - is not affected by parental education. As a result, for low wage individuals, for whom the opportunity cost of time is low, children are cheap and the relative price of child quantity increases with the wage rate. Therefore, this assumption generates a comparative advantage for the poor in child quantity, whereas high wage (educated) individuals have a comparative advantage in raising quality children. Consistent with the well-known evidence, poor households thus choose relatively high fertility rates with relatively low investment in their offspring's education; and therefore, their offspring are poor as well. In contrast, high-income families choose low fertility rates with high investment in education, and therefore, high income persists in the dynasty.

Indeed, Rosenzweig and Wolpin (1980) and Hanushek, (1992) provide evidence that there exists a trade-off between quantity and quality of children. Altonji and Dunn (1996) and Ahituv (2001) find that parents' education positively affects children's education, and Kremer and Chen (1999) show that fertility is negatively correlated with education. In addition, Lam and Duryea (1999) find a strong negative effect of women's schooling on fertility, a strong positive effect of parental schooling on children's schooling, and argue that the effects of schooling on fertility works primarily through increased investment in child quality.¹ Behrman et al (1999) find that increases in the schooling of women enhance the human capital of the next generation, and, consistent with the underlying mechanism developed here, they argue that a component of the significant and positive relationship between maternal literacy and child schooling reflects the productivity effect

¹Consistent with the paper's key assumption, Lam and Duryea (1999) argue that mothers with more schooling, despite the increase in their market wages, do not increase their labor supply because of their higher productivity in producing well-educated children. In addition, Psacharopoulos et al (1996) find that teachers in Latin American countries are rewarded for their education similarly to the average reward in the economy.

of home teaching.²

In my basic model, dynasties within a country can converge to one of the two equilibria; either a low education - high fertility equilibrium, or a high education - low fertility equilibrium. An extension of this basic model captures the diluting effect of fertility on the accumulation of physical capital, amplifying the effect of quality choice on income per capita. Consistent with the negative cross-country relationship between fertility and growth and the positive cross-country relationship between education and growth, countries can converge to two different levels of income per capita. The high-income steady state, in contrast to the low-income steady state, is characterized by high levels of physical and human capital per capita and low rates of fertility. Ahituv (2001) finds that in low-income countries - with GDP of less than \$1,000 per capita - the average fertility rate was 6.5 children per woman in 1965, as oppose to only 3.7 in the high-income group with per capita GDP above \$2,500 . During the same period only 6.0% of the relevant school age children were enrolled in secondary school in low-income countries, compared to 49.2% in the high-income countries. The picture in 1985 was very similar; the average fertility rate in the low-income countries was still above six, and in the high-income group the average has declined to 2.4. Moreover, Barro (1991) shows that economic growth is positively related to human capital, and countries with higher human capital also have lower fertility rates and higher ratios of physical investment to GDP, and Kelley and Schmidt (1995) show that population growth has a negative impact on economic growth.

Economic growth models of fertility designed to explain the possibility of multiple equilibria - a poverty trap equilibrium and a high income equilibrium - go back to Nelson

²See also the survey of the evidence by Schultz (1993) and by Strauss and Thomas (1995). They show that women's education is strongly negatively correlated with fertility and positively correlated with child quality (education and health), and that fathers' education has a similar effect but of less magnitude. Behrman and Rosenzweig (2001), however, find a marginally negative rather than a significantly positive coefficient for mother's schooling in the determination of child schooling once ability is controlled. They find that increased maternal schooling leads to reduced home time for mothers, implying that the increase in mothers wages more than offsets their increased productivity in child quality. They do find, however, a positive and statistically significant coefficient of father's schooling on children's schooling. Finally, they argue that their findings must be interpreted with care because it is possible that increased mother's schooling leads to increased child schooling elsewhere as shown by Behrman, et al (1999).

(1956). He shows that in an environment in which fertility and saving rates increase with income, an underdevelopment trap with low savings is plausible. In this trap, even if capital is accumulated, population rises at an equal rate. More recently, Becker, Murphy and Tamura (1990), have developed a representative agent model in which individuals face the trade-off between the quality and quantity of their offspring. Their model generates multiple steady states consistent with the cross-country relationship between fertility, education and growth. The source of multiplicity of equilibria in their model is the assumption that the return to education is lower in poor economies. In particular, they assume that the return to human capital increases with the aggregate level of education in society. Their approach suffers from both theoretical and empirical limitations. The poverty trap equilibrium is a result of a market failure in a framework in which education has a positive externality, where existing evidence contradicts the underlying assumption of increasing returns to education with income.³ Furthermore, while fertility decisions in the analysis of Becker, Murphy and Tamura (1990) may amplify the negative impact of low education investment on income per capita, they are not the source of multiple equilibria.⁴

In this paper, in contrast, a simple dynamical system generates multiple steady states that emerge from the comparative advantage of educated workers in the production of educated children. The model is based neither on restrictive assumptions concerning preferences, or the return to human capital, nor on any non-convexity in the production process.

The model generates a testable prediction regarding the correlation of education and income within a dynasty. Individuals with a high level of human capital would invest highly in their offspring's education, even in poor economies. In contrast, the model developed by Becker, Murphy and Tamura (1990) implies that in poor economies, where human capital is scarce, the investment in human capital is low due to its low return, regardless

³See, for instance, Psacharopoulos (1994), Angrist (1995) and Acemoglu and Angrist (2000).

⁴Tamura (1996) also generates two development regimes based on a rising rate of return to human capital investment and a conditional external effect in human capital investment. See also Barro and Becker (1989) who offer an explanation for the negative cross-section relation between income and population growth based on the effect of the trade-off between the size of the real transfer to each child and the number of children on the unique steady state of their model.

of the parents' level of human capital. That is, their model implies that in poor economies all families eventually converge to a low-education equilibrium and in rich economies all families eventually converge to a high-education equilibrium. In contrast, according to my model, poverty can persist in wealthy countries, and wealthy (educated) individuals can exist in the long run in the poor countries.

The micro-foundations of this paper follow the concept of a trade-off between child quality and child quantity analyzed by Becker and Lewis (1973). Their principal observation is that the cost of an additional child increases with the desired level of child quality. Therefore, under the assumption that both child quantity and child quality are normal goods, a rise in income has two opposite effects on the quantity of children. While the increase in income has a direct positive effect on the quantity of children, it also increases their quality and thus their cost, negatively affecting their quantity. Becker and Lewis show, therefore, that in spite of the normality of the demand for children, if preferences are non-homothetic, the observed relationship between the quantity of children and income can be negative.

In this paper, as in Becker and Lewis (1973), the cost of an additional child increases with the desired level of child quality; and the cost of quality increases with the number of children, generating a non-convex budget set. However, in contrast to Becker and Lewis, here the key assumption is that individual's productivity in educating children increases with their own human capital, whereas the fraction of the individual's time endowment that is required in order to raise a child, regardless of quality - the quantity cost - is equal across all individuals. For instance, while all individuals are equally able in feeding a child, individuals' effectiveness in helping their children with homework increases with their own level of education. This assumption implies that the ratio between the price of quantity and the price of quality increases with individual's wage which generates a comparative advantage for the poor in child quantity, and a comparative advantage for the wealthy (educated) in raising quality children. The impact of changes in wages is amplified by the non-convexity of the budget set bringing about the negative correlation between income and fertility and multiple equilibria.

On a more intuitive level, the mechanism is based on a "multiplier effect". A decline in parental education, and hence in their income, leaves less resources for children's

education. The increased fertility, due to the lower time cost, further reduces recourses for education, which are in addition divided between more children. Therefore, differences in parents education can be amplified when it comes to differences in offspring education, generating multiple equilibria. Interestingly, the poor in each generation can choose to invest highly in education, on the account of their fertility rate and allow their offspring to escape the poverty cycle. However, they prefer not to do so. The endogenous fertility framework, with a trade-off between quality and quantity, raises an inherent conflict of interest between parents and offspring. Parents care about both the quality of each child and the quantity of children, whereas children prefer less siblings and more human capital. Since parents make the decisions in the household, poverty may persist.

This paper also offers an explanation for the impact of income inequality on economic growth.⁵ This line of research received a lot of attention in the last decade. Due to mobility constraints, poor dynasties remain poor, leading economies to an underdevelopment trap. Banerjee and Newman (1993), Galor and Zeira (1993), Benabou (1996), Durlauf (1996), Piketty (1997), Maoz and Moav (1999), Ghatak and Jiang (2000) and Mookherjee and Ray (2000), among others, show that credit constraints combined with non-convexities in the technology prevent investment by the poor, generating a persistence of poverty.⁶ In this paper the dynamical system generates a poverty trap along with a high-income (high human capital) equilibrium. Poor dynasties with income below a threshold level converge to the low income steady state, whereas dynasties with income above the threshold level converge to a high income steady state. Therefore, if the initial average income in society is above the threshold, then in a more equal society, more individuals are above the threshold and more dynasties converge to the high steady state. Hence, consistent with evidence offered by Perotti (1996) and Barro (1999), inequality negatively affects economic growth

⁵The impact of economic growth on income inequality, as recently studied by Acemoglu (1998), Galor and Moav (2000a), Gould, Moav Weinberg (2001), Eicher and Gracia-Penalosa (2001), and Rubinstein and Tsiddon (1998) among many others, is not the focus of this paper.

⁶In the model developed by Piketty (1997), the effort level, rather than capital investment, is indivisible. Mookherjee and Ray (2000) show that while inequality persists irrespective of the divisibility of human capital, the multiplicity of steady states requires indivisibilities in the return to education. See also Moav (2002), where increasing saving rates with income, replace the role of non-convexities in the technology in generating multiple steady states.

via its interaction with fertility choice.⁷ Moreover, in contrast to the existing literature, the result of a long run impact of the initial wealth distribution is generated in spite of convex human capital and final good production technologies and convex homothetic preferences.⁸

Recent literature on population and growth offers explanations for a demographic transition and a take-off from economic stagnation to sustained economic growth. Following the insight of Schultz (1975), Galor and Weil (2000) assume that a rise in the rate of technological progress increases the rate of return to human capital, inducing parents to substitute child quality for child quantity.⁹ They show that the positive interaction between population and technology gradually increased the rate of technological progress, inducing investment in human capital that lead to a demographic transition and sustained growth. Galor and Moav (2000*b*) develop a unified evolutionary growth theory that captures the interplay between the evolution of mankind and economic growth. They suggest that prolonged economic stagnation, prior to the transition to sustained growth, stimulated natural selection, that shaped the evolution of the human species and eventually brought about the take-off from stagnation to sustained growth.¹⁰ In Galor and Weil (2000) as well as in Galor and Moav (2000*b*), technological progress brings about a demographic transition and an escape from a temporary poverty trap steady state. Here, in contrast, since labor is a factor of production of human capital, the cost of education increases with wages and hence the model's multiple steady states are robust to technical progress. Thus, while technology is (at least partly) available to less developed economies,¹¹ the model offers an explanation for the observed high fertility and the persistence of poverty in many countries around the world.

⁷Banerjee and Duflo (1999) however, argue that this is a result of a "Latin American effect."

⁸See also Hazan and Berdugo (2002) who study the relationship between child labor, fertility and growth, and Kremer and Chen (1999) who study the effect of income inequality on economic growth in an endogenous fertility framework. In addition Veloso (1999) analyzes the effect of different compositions of wealth and human capital on education.

⁹See Galor and Moav (2000*a*) who offer an explanation for the effect of technological progress on the return to human capital.

¹⁰Long-run growth models with endogenous population also include the work of Kremer (1993), Galor and Weil (1996), Dahan and Tsiddon (1998), Morand (1999), Hansen and Prescott (2000), Jones (2000) and Lagerloff (2000) among others.

¹¹See Acemoglu and Zilibotti (2001).

2 The Basic Model

Consider an overlapping generation economy in which activity extends over infinite discrete time. In every period the economy produces a single homogenous good, in a constant-returns-to-scale technology, using human capital as a single input. The supply of human capital is determined by households' decisions in the preceding period regarding the number of their children and the level of education investment in each child.

2.1 Individuals

In each period a generation of individuals, who each has a single parent, is born. Individuals live two periods: In childhood they acquire human capital; in adulthood they are endowed with one unit of time, which they allocate between child rearing and participation in the labor force .

The preferences of members of generation t (born in $t - 1$) are defined over consumption as well as over the quality and quantity of their children, where quality is measured by their offspring's full income (potential income). Preferences are represented by the utility function,

$$u_t^i = (1 - \beta) \log c_t^i + \beta[\log n_t^i + \theta \log wh_{t+1}^i], \quad (1)$$

where $\beta \in (0, 1)$ captures the relative weight given to children (quality as well as quantity) and $\theta > 0$ captures the relative weight given to child quality in the utility function. c_t^i is the consumption in the household of a member i of generation t , n_t^i is the number of children in this household, w is the wage rate per efficiency unit of labor,¹² and h_{t+1}^i is the level of human capital of each child measured in efficiency units.¹³

¹²As follows from the assumption of a single production factor in a CRS technology, the wage rate per efficiency unit of labor, w , is constant over time. This assumption is relaxed in the next section where both physical and human capital are employed in the production process. Including the wage rate in the utility function implies that parents care about offspring income. Removing the wage rate from the utility will allow for a different interpretation - that schooling is valued for its own sake - with no impact on the model's results.

¹³Defining preferences over the quality of each child, rather than the average quality, and thus, implicitly restricting parents from treating children unequally is not a binding constraint in the context of this model. Children are identical with respect to their human capital production technology, which is characterized

2.2 The Formation of Human Capital

In the first period of their lives individuals devote their entire time for the acquisition of human capital (measured in efficiency units of labor). The acquired level of human capital increases if their time investment is supplemented by investment in education. However, even in the absence of investment in education, individuals acquire one efficiency unit of labor - basic skills. The level of investment in education of individual i born in period t , e_{t+1}^i , is measured in efficiency units of labor, capturing the model's key assumption that individuals' productivity as teachers increases with their own human capital. Since e_{t+1}^i is measured in efficiency units of labor, the real cost of the investment in education is we_{t+1}^i , whether this is viewed as a direct cost of hiring a teacher or an opportunity cost of teaching one's own children.¹⁴ The resulting number of efficiency units of labor of that individual in period $t + 1$, h_{t+1}^i , is a strictly increasing, strictly concave function of investment in education in period t , e_{t+1}^i ,

$$h_{t+1}^i = h(e_{t+1}^i), \quad (2)$$

where $h(0) = 1$, $\lim_{e \rightarrow 0^+} h'(e^i) = \gamma$, $\lim_{e \rightarrow \infty} h(e^i) > 1/\tau\theta\gamma$,¹⁵ and $\lim_{e \rightarrow \infty} h'(e^i) = 0$. The assumption that the slope of the production function of human capital is finite at the origin along with the ability of individuals to supply some minimal level of labor regardless of the investment in human capital (beyond time), assure that under some conditions raising quality children is not optimal.¹⁶

by decreasing returns, and therefore, parents would provide equal education to all their children under both specifications.

¹⁴The specific formulation implies that human capital increases the productivity in educating exactly as much as it increases the productivity in producing the final good. This is just a simplifying assumption that enables circumventing the explicit modeling of the market for teachers. Alternatively, individuals within some range of human capital would have a comparative advantage as teachers, and those individuals who have a comparative advantage in producing the final good would hire teachers for their children. This has no qualitative impact on the analysis since individuals under the existing structure are indifferent between buying education or providing it directly.

¹⁵where τ , formally defined below, is the time cost for raising a child.

¹⁶The Inada conditions are typically designed to simplify the exposition by avoiding a corner solution, but they are surely not realistic assumptions.

2.3 Budget Constraint

Let τ be the minimum time cost required for raising a child; additional time allocated to children positively affect their quality. That is, τ is the fraction of the individual's unit of time endowment that is required in order to raise a child, regardless of quality. Following Becker and Lewis (1973), Rosenzweig and Wolpin (1980) and Galor and Weil (2000), it is assumed for the sake of simplicity that the "quantity cost" per child does not vary with family size. The economic force behind the fertility gap between rich and poor is the lower quantity cost faced by the poor. Therefore, incorporating a range of decreasing quantity cost would increase the marginal cost difference faced by poor and rich, amplifying the fertility gap, and thus strengthening the paper's results. Increasing returns to investment in quality would work in the opposite direction, but, up to a limit, should not have a qualitative effect on the model's results.¹⁷ As will become apparent, fertility rates are bounded from above by β/τ , therefore it is assumed that $\tau < \beta$. It is further assumed that τ is sufficiently small, so that individuals with a low level of human capital choose the corner solution of zero investment in child education,

$$\tau < 1/\theta\gamma. \tag{A1}$$

Consider an adult member i of generation t who is endowed with h_t^i efficiency units of labor at time t , where $h_t^i = h(e_t^i)$. Full income, wh_t^i , is divided between expenditure on child rearing (quantity as well as quality) and consumption, c_t^i . The (opportunity) cost of raising each child, regardless of quality, is equal to $w\tau h_t^i$, and the cost of quality of each child is equal to we_{t+1}^i . The cost of raising n_t^i children, with an education level of e_{t+1}^i is given therefore, by $n_t^i[w\tau h_t^i + we_{t+1}^i]$, and the individual faces the budget constraint

$$n_t^i w[\tau h_t^i + e_{t+1}^i] + c_t^i \leq wh_t^i. \tag{3}$$

As captured by the budget constraint, given in equation (3), the cost of child quantity, $w\tau h_t^i$, in contrast to the cost of child quality, we_{t+1}^i , increases with the level of human capital of the individual, h_t^i . This is a result of the assumption that individuals' productivity as educators, in contrast to their productivity in child quantity, increases with their own human capital.

¹⁷It is implicitly assumed that the time cost of raising a child can not be reduced by childcare employment.

2.4 Optimization

Members of generation t choose the number and quality of their children, and their own consumption, so as to maximize their utility function, subject to the budget constraint. It follows from the optimization that consumption is given by,

$$c_t^i = (1 - \beta)wh_t^i = (1 - \beta)wh(e_t^i). \quad (4)$$

That is, a fraction $1 - \beta$ of full income is devoted for consumption and hence a fraction β of full income is devoted to child rearing in terms of quality and quantity. Furthermore, the optimization with respect to child quality, e_{t+1}^i , is given by,¹⁸

$$\frac{\theta h'(e_{t+1}^i)}{h(e_{t+1}^i)} - \frac{1}{\tau h(e_t^i) + e_{t+1}} \begin{cases} \leq 0 & \text{if } e_{t+1}^i = 0; \\ = 0 & \text{if } e_{t+1}^i > 0, \end{cases} \quad (5)$$

where it is assumed that θ is sufficiently small such that the optimal investment, e_{t+1}^i is not approaching infinity.¹⁹ In particular it is assumed that,

$$F''(e_{t+1}^i) < 0, \quad (A2)$$

where $F(e_{t+1}^i) \equiv [h(e_{t+1}^i)]^\theta$.

Lemma 1 *Under Assumptions A1 and A2 there exists a single valued function $\phi(e_t^i)$ such that,*

$$e_{t+1}^i = \phi(e_t^i) \begin{cases} = 0 & \text{if } e_t^i \leq \hat{e} \\ > 0 & \text{if } e_t^i > \hat{e} \end{cases}$$

¹⁸Which is consistent with the standard condition of setting the marginal rate of substitution between quality and quantity equal to the price ratio (or larger in case of a corner solution),

$$\frac{h(e_{t+1}^i)}{\theta n_t^i} - \frac{\tau h(e_t^i) + e_{t+1}}{n_t^i/h'(e_{t+1}^i)} \begin{cases} \geq 0 & \text{if } e_{t+1}^i = 0; \\ = 0 & \text{if } e_{t+1}^i > 0. \end{cases}$$

where $h(e_{t+1}^i)/\theta n_t^i$ is the marginal rate of substitution between quality and quantity, $(\tau h(e_t^i) + e_{t+1})w$ is the cost of an additional child and $wn_t^i/h'(e_{t+1}^i)$ is the marginal cost of children's quality (human capital). The wage rate has no bearing on the optimization since both the quality and quantity cost are its products.

¹⁹Alternatively, a strictly positive lower limit to the number of children can be postulated. This is a reasonable restriction since children come in natural numbers and zero is ruled out by the optimization. Under this restriction, there is a threshold level of education, above which individuals choose a corner solution of having the lowest possible number of children with high investment in education, with no qualitative impact on the model's results.

where $\phi'(e_t^i) > 0$ for $e_t^i > \hat{e}$ and $\hat{e} > 0$ is unique and given by $1/\theta\gamma = \tau h(\hat{e})$.

Proof.

Define $G(e_{t+1}) \equiv h(e_{t+1})/\theta h'(e_{t+1})$. It follows from (5) that

$$G(e_{t+1}^i) \geq \tau h(e_t^i) + e_{t+1}^i. \quad (6)$$

As follows from A1, $\partial G(e_{t+1}^i)/\partial e_t^i = 0$, $\partial[\tau h(e_t^i) + e_{t+1}^i]/\partial e_t^i > 0$, and $G(0) = 1/\theta\gamma > \tau h(0) = \tau$, and as follows from the properties of (2) $G(0) = 1/\theta\gamma < \lim_{e \rightarrow \infty} \tau h(e_t^i)$, and $h'(e_t^i) > 0$. Therefore, as follows from the intermediate value theorem, there exists a unique $e_t^i = \hat{e} > 0$, given by $1/\theta\gamma = \tau h(\hat{e})$, such that $G(0) = \tau h(\hat{e})$ and $G(0) > (<)\tau h(e_t^i)$ for all $e_t^i < (>)\hat{e}$ implying that $\phi(e_t^i) = 0$ for all $e_t^i \leq \hat{e}$ and $\phi(e_t^i) > 0$ otherwise.

As follows from implicit differentiation of the first order condition of the maximization problem as given by (5), noting that under A2 second order condition hold for a maximum, $\phi(e_t^i)$ is single valued and $\phi'(e_t^i) > 0$. \square

It follows from Lemma 1, and equations (3) and (4) that the number of children of a member i of generation t , n_t^i is given by

$$n_t^i = n(e_t^i) = \begin{cases} \beta/\tau & \text{if } e_t^i \leq \hat{e}; \\ \beta h(e_t^i)/[\tau h(e_t^i) + \phi(e_t^i)] & \text{if } e_t^i > \hat{e}. \end{cases} \quad (7)$$

where $\beta/\tau \geq \beta h(e_t^i)/[\tau h(e_t^i) + \phi(e_t^i)]$. That is, fertility rates among low education individuals ($e_t^i \leq \hat{e}$), who choose not to invest in the quality of their children, are higher than those among individuals with higher education levels who choose to invest in the education of their children. However, depending on the properties of the human capital production function, fertility rates may decrease or increase with the level of human capital for $e_t^i > \hat{e}$.

The effect of non-wage income is straightforward. A change in income, which is not related to a change in individuals' human capital, has no effect on the quality-quantity price ratio, nor on the consumption price. Therefore, due to the assumption that preferences are homothetic, it will give rise to a proportional increase in resources allocated to consumption, quality and quantity. That is, the education of each child is not effected, implying that equation (5) and Lemma 1 hold for the general case where individuals' income is not restricted to labor.

2.5 The Dynamical System

Figure 1 depicts the dynamical system, $\phi(e_t^i)$. Assumption A1 assures the existence of a low education steady state, $\phi(0) = 0$. However, in order for $\phi(e_t^i)$ to generate a high education steady state there must exist a range of e_t^i in which $\phi(e_t^i)$ is sufficiently sensitive to changes in e_t^i . That is, a range in which small changes in the parent's education bring about large changes in the offspring education and as a result $\phi(e_t^i) > e_t^i$ for some e_t^i .

Proposition 1 *There exist a human capital production function, that satisfies Assumptions A1 and A2, and the properties of $h(e_t^i)$, in equation (2), such that the dynamical system, $\phi(e_t^i)$, is characterized by multiple steady states.*

Proof.

The proof follows from an example. Consider the following human capital production function,

$$h_{t+1}^i = h(e_{t+1}^i) = \begin{cases} 1 + \gamma e_{t+1}^i & \text{if } e_{t+1}^i < \bar{e}; \\ 1 + \gamma \bar{e} & \text{if } e_{t+1}^i \geq \bar{e}. \end{cases} \quad (8)$$

where $\bar{e} > (1 - \tau\gamma)/\tau\gamma^2$. Hence, it follows from the optimization problem of a member i of generation t , that is endowed with $h(e_t^i)$ efficiency units of human capital that

$$e_{t+1}^i = \bar{\phi}(e_t^i) \begin{cases} = 0 & \text{if } e_t^i < \hat{e}; \\ \in [0, \bar{e}] & \text{if } e_t^i = \hat{e}; \\ = \bar{e} & \text{if } e_t^i > \hat{e}, \end{cases} \quad (9)$$

where as follows from (8) and Assumption A1, $\hat{e} = (1 - \tau\gamma)/\tau\gamma^2 > 0$ and therefore $\bar{e} > \hat{e}$. The dynamical system $\bar{\phi}(e_t^i)$ is characterized by two stable steady states, \bar{e} and 0, and an unstable steady state \hat{e} , which is the threshold level of education. The lemma thus follows from continuity considerations.²⁰ \square

²⁰ While the model is not designed to perform calibrations, a numerical illustration reveals that reasonable parameters, in particular the lifetime dollar return to each dollar invested in education, as given by γ , (see Psacharopoulos, 1994) can generate multiple equilibria. In particular, for $\gamma = 2$, $\theta = 1$, $\tau = 0.15$, $\beta = 1/2$ and $\bar{e} = 1.5$, earnings at the high income equilibrium are 4 times higher than earnings at the low income,

2.6 Steady States and Implications of the Basic Model

As depicted in Figure 1, a dynasty i , with a given initial level of education, e_0^i , will converge to a high education, high income, and low fertility steady state if $e_0^i > e^T$.²¹ Otherwise, the dynasty will converge to a low income, low human capital, and high fertility steady state. The mechanism generating multiple steady states is based on the effect of parental education on child quantity cost. The lower is parents' education, that is the cheaper is parents' time, the cheaper are children and the parents' choice shifts to higher fertility rates and lower investment in offspring's human capital. Note that a decline in parental education, and hence in their income, leaves less resources for children's education even if their number is unchanged. The increased fertility further reduces investment in education which is, in addition, divided between more children. Therefore, consistent with Proposition 1, differences in parental education can be amplified when it comes to differences in offspring education.

The dynamical system generates predictions on the effect of income inequality on economic growth. If initial average income and the corresponding average level of human capital are above the threshold, e^T , then growth in the economy will be higher the more equal is society, since more dynasties converge to the high steady state. Hence, consistent with evidence (Perotti 1996; Barro, 1999), income equality generates higher output via its interaction with fertility choice.

The effect of changes in the quantity cost parameter, τ , on the dynamical system and its steady states follows from equation (5) and Lemma 1. An increase in τ increases the relative cost of quantity, inducing a shift to child quality. Hence, it reduces the level and fertility rates 1.9 children per household (of two parents) in the high income and 6.6 at the low. Note that multiplicity will hold for any set of parameters restricted by A1, A2 and $\bar{e} > \hat{e}$. An alternative example, more closely related to the commonly assumed Cobb-Douglas human capital production (see, for instance, Heckman et al, 1998), satisfies concavity. Suppose $h(e) = \alpha^{-\alpha\gamma}(\alpha + e)^{\alpha\gamma}$. It is straightforward to verify that the properties of (2) are satisfied. Under this specification multiplicity arises if, for instance, $\gamma = 2$, $\theta = 1$, $\tau = 0.15$, $\beta = 1/2$, $\alpha = 0.45$, and an upper limit to e of 2.5 or higher is imposed. Note that the simple Cobb-Douglas function implies that, the commonly imposed, though unrealistic, Inada conditions hold, and thus will be inconsistent with this paper's assumptions.

²¹Note that the threshold level of education that is sufficient in order to converge to the high human capital steady state, e^T , is higher than the threshold level that generates a strictly positive investment in human capital, \hat{e} .

of the threshold below which individuals choose not to purchase any education for their offspring, \hat{e} , and increases the level of human capital, $\phi(e_t^i)$, above \hat{e} . This implies that the dynamical system depicted in Figure 1 shifts upward, the threshold level, e^T , declines, and the high income steady state, \bar{e} , increases.

The effect of changes in the cost of education is not so straightforward. The analysis of public schooling on fertility and education, which is the most relevant question regarding policy implications, follows.²² Assume that the government supplies free of charge schooling at a level e_t^g , which is financed by foreign aid or taxation.²³ It follows from (5), that if $\phi(e_t^i) < e_t^g$ parental investment in education would be zero, or otherwise it would equal $\phi(e_t^i) - e_t^g$. Public schooling has, therefore, a positive effect on offspring's level of education (for all, i such that $\phi(e_t^i) < e_t^g$), but it also reduces parental expenditure on education (for all i such that $\phi(e_t^i) > 0$) and therefore gives rise to a reallocation of resources for increased fertility. In the long-run however, if $e_t^g > e^T$, public schooling will shift dynasties to a path of increased education and income and, as long as $e_t^g < \bar{e}$, reduced fertility.

3 The Extended Model

In this section, endogenous physical capital accumulation is introduced, allowing the model to capture the diluting effect of fertility on capital per worker, which amplifies the effect of quality choice on economic growth. This section therefore, focuses on the impact of fertility on the economy rather than on poverty within an economy, offering an explanation for cross-country income differences and club convergence.

The basic model is enriched by allowing individuals to bequeath capital to their offspring, in addition to the investment in child quality. Final output is thus produced by two factors of production: physical and human capital. The aggregate supply of production factors is determined by individuals' choice of physical capital bequest, educational expenditures, and fertility in the previous period. For the sake of simplicity, the parameter θ in the utility function is set to one and, given the section's focus, the analysis performed

²²The analysis abstracts from the potential effect of public schooling on the quantity cost.

²³An indirect consumption tax, for instance, will not have an impact on the quality and quantity choice of individuals.

assumes homogenous individuals.

3.1 Production

Production occurs within a period according to a neoclassical, constant-returns-to-scale, Cobb-Douglas production technology. The output produced at time t , Y_t , is given by

$$Y_t = F(K_t, H_t) \equiv H_t f(k_t) = H_t A k_t^\alpha; \quad k_t \equiv K_t/H_t, \quad (10)$$

where K_t and H_t are physical and human capital employed in production in period t .

Producers operate in a perfectly competitive environment and therefore production factors are paid according to their marginal products,

$$\begin{aligned} w_t &= (1 - \alpha) A k_t^{\alpha-1} \equiv w(k_t); \\ r_t &= \alpha A k_t^{\alpha-1} \equiv r(k_t), \end{aligned} \quad (11)$$

where w_t is the wage rate per efficiency unit of labor in time t , and r_t is the capital rate of return. For sake of simplicity, physical capital is assumed to fully depreciate at the end of each period.

3.2 Optimization

Members of generation t choose the number, n_t , and quality, e_{t+1} , of their children, the quantity of physical capital they transfer to each child, s_{t+1} , and the household consumption, c_t , so as to maximize their utility function,

$$u_t = (1 - \beta) \log c_t + \beta [\log n_t + \log(w_{t+1} h_{t+1} + r_{t+1} s_{t+1})], \quad (12)$$

subject to the human capital production technology, $h_{t+1} = h(e_{t+1})$, and the budget constraint,

$$n_t [w_t \tau h_t + w_t e_{t+1} + s_{t+1}] + c_t \leq w_t h_t + r_t s_t, \quad (13)$$

where $w_t h_t + r_t s_t$ is full income of each individual in period t , and $h_t = h(e_t)$. Defining preferences over the income of each child implies that investment in education is optimal in the standard form of equalizing the marginal returns to human and physical capital in

any interior solution. It follows from the optimization that individuals consume a fraction $(1 - \beta)$ of full income,

$$c_t = (1 - \beta)[w_t h_t + r_t s_t],$$

and a fraction β of full income is devoted to children's quality, quantity, and capital transfers. In particular, the optimization with respect to capital transfers, s_{t+1} , is given by,

$$\frac{h(e_{t+1})w_{t+1}}{(\tau h_t + e_{t+1})w_t} - r_{t+1} \begin{cases} > 0 & s_{t+1} = 0; \\ = 0 & s_{t+1} \in [0, \infty); \\ < 0 & s_{t+1} \rightarrow \infty.^{24} \end{cases} \quad (14)$$

In equilibrium, however, since individuals are identical within each generation, it follows from (14) that

$$\frac{h(e_{t+1})w_{t+1}}{(\tau h_t + e_{t+1})w_t} = r_{t+1}. \quad (15)$$

Otherwise, if the left-hand side is larger (smaller), there is no physical (human) capital in period $t + 1$, and the left-hand side is smaller (larger) in contradiction. Given (15), it follows from the optimization with respect to e_{t+1} that²⁵

$$\frac{h(e_{t+1})}{\tau h_t + e_{t+1}} - h'(e_{t+1}) \begin{cases} \geq 0 & \text{if } e_{t+1} = 0; \\ = 0 & \text{if } e_{t+1} > 0, \end{cases} \quad (16)$$

which is the condition derived in equation (5) in the basic model section for $\theta = 1$. That is, for $\theta = 1$, the introduction of endogenous wages and capital bequest did not alter the optimal level of education parents choose for each child and therefore, as follows from Lemma 1, the dynamical system governing the evolution of education is given by $e_{t+1} = \phi(e_t)$.²⁶

3.3 The Dynamical System

It follows from Lemma 1 and equations (11), (15) and (16) that the dynamical system is uniquely determined by the sequence $\{k_t, e_t\}_{t=0}^{\infty}$ such that

²⁵Individuals' optimization, regardless of the homogeneity assumption and therefore regardless of equation (15), assures efficient investment in education: $r_{t+1} - \frac{h'(e_{t+1})w_{t+1}}{w_t} \begin{cases} \geq 0 & \text{if } e_{t+1} = 0 \\ = 0 & \text{if } e_{t+1} > 0 \end{cases}$

²⁶Resources allocated to capital transfers come on the account of the fertility rate.

$$\begin{cases} e_{t+1} = \phi(e_t); \\ k_{t+1} = \psi(e_t, k_t) = \alpha A k_t^\alpha [\tau h(e_t) + \phi(e_t)] / h(\phi(e_t)), \end{cases} \quad (17)$$

where k_0 and e_0 are given. Note that k_t is the physical human capital ratio, and that physical capital per worker is equal to $h(e_t)k_t = s_t > k_t$. Output per worker, y_t , as follows from (10), is therefore uniquely determined by the dynamical system,

$$y_t = h(e_t)A k_t^\alpha = A h_t^{1-\alpha} s_t^\alpha.$$

The kk Locus

Let kk be the locus of all pairs (k_t, e_t) such that physical capital per efficiency unit of labor, k_t , is in a steady-state: $kk \equiv \{(k_t, e_t) : k_{t+1} = k_t\}$. As follows from (17) there exists a function

$$k^{kk}(e_t) = \left(\frac{\alpha A [\tau h(e_t) + \phi(e_t)]}{h(\phi(e_t))} \right)^{1/(1-\alpha)}, \quad (18)$$

such that if $k_t = k^{kk}(e_t)$, then $k_{t+1} = \psi(e_t, k_t) = k_t$, that is the kk Locus consists of all the pairs $(k^{kk}(e_t), e_t)$.

Lemma 2 $dk^{kk}(e_t)/de_t > 0$. That is, as depicted in Figure 2, the kk Locus is strictly increasing in the plan (e_t, k_t) .

Proof.

For $e_t \leq \hat{e}$, as follows from Lemma 1, $\phi(e_t) = 0$, and therefore, $h(\phi(e_t)) = 1$, and since $h'(e_t) > 0$, it follows that $dk^{kk}(e_t)/de_t > 0$ for $e_t \leq \hat{e}$.

For $e_t > \hat{e}$, it follows from (16) and Lemma 1 that $h(\phi(e_t))/[\tau h(e_t) + \phi(e_t)] = h'(\phi(e_t))$. Furthermore, as established in Lemma 1 $\phi'(e_t) > 0$ for $e_t > \hat{e}$, and since $h''(e_t) < 0$ it follows that $h(\phi(e_t))/[\tau h(e_t) + \phi(e_t)]$ is strictly decreasing with e_t , and therefore $dk^{kk}(e_t)/de_t > 0$ for $e_t > \hat{e}$. \square

The ee Locus

Let ee be the locus of all pairs (k_t, e_t) such that the level of investment in human capital per capita, e_t , is in a steady-state: $ee \equiv \{(k_t, e_t) : e_{t+1} = e_t\}$. However, as follows from (17), the evolution of e_t is independent of the evolution of the physical human capital ratio,

and hence $ee \equiv \{e_t : \phi(e_t) = e_t\}$. As follows from the properties of $\phi(e_t)$ and as depicted in Figure 1, $e_t = \phi(e_t)$ for $e_t = 0$, $e_t = e^T$ and $e_t = \bar{e}$, therefore the ee locus consists of three vertical lines in Figure 2: $e = 0$, $e = e^T$ and $e = \bar{e}$.

The evolution of e_t as follows from (17) and depicted in Figure 1 is given by $e_t = \phi(e_t)$. Therefore, as depicted in figure 2, $e_{t+1} > e_t$ for all $e_t \in (e^T, \bar{e})$, whereas $e_{t+1} < e_t$ for all $e_t < e^T$ and all $e_t > \bar{e}$. The dynamics of k_t follow from (17). As depicted in Figure 2, $k_{t+1} > k_t$ for all $k_t < k^{kk}(e_t)$, and vice versa. Hence, k_t is increasing below the kk Locus and decreasing above it.

3.4 Steady States and Implications of the Extended Model

The model generates two locally stable steady states. If the initial level of education is above the threshold level, $e_0 > e^T$, the economy converges monotonically to the high capital labor ratio, high education steady state, characterized by a low fertility rate. If however $e_0 < e^T$, the economy converges to the low capital labor ratio, low education steady state - the poverty trap - that is characterized by a high fertility rate.²⁷

It is interesting to note that in the context of a closed economy, the low output steady state is not a result of capital market imperfections nor any other market failure, and resource allocation is dynamically efficient - the marginal return to education is not higher than the marginal return to physical capital. Hence, even if individuals could borrow to finance their own education they choose not to do so. A shift of the economy from the low to the high output steady state, can be achieved only if one generation gives up child quantity in favor of child quality, and suffers from a utility cost. Of course, if countries differ from each other, in particular, if some economies are in the high output steady state, than the low output steady state is an outcome of imperfection in international capital markets (taken to the extreme of closed economies in the model).

As argued previously, the mechanism generating multiple steady states is based on the effect of parental education on child quantity cost. Lower parental education, implies lower time cost and therefore cheaper children. Hence, lower parental education brings about a reallocation of resources from child quality to quantity. The effect of quality

²⁷In the poverty trap the level of education is zero and as follows from (18) the capital labor ratio is given by $(\alpha A\tau)^{1/(1-\alpha)}$.

choice on output per capita is amplified in the extended model by its consequence on fertility and the diluting effect fertility has on capital accumulation. In the poverty trap therefore, the high fertility rates yields a low capital labor ratio.

An increase in the quantity time cost, τ , reduces the threshold for purchasing education, \hat{e} , and raises the level of education, $\phi(e_t)$, above \hat{e} . This implies that $\phi(e_t)$, depicted in Figure 1, shifts upward, the threshold level, e^T , declines, and the high income steady state, \bar{e} , increases. Therefore, the ee Locus, depicted in Figure 2, shifts accordingly, that is, the vertical line at \bar{e} shifts to the right, while the vertical threshold line at e^T shifts to the left. Furthermore, since $\phi(e_t)$ increases with τ for $e > \hat{e}$, it follows from Lemma 1, the concavity of $h(e_t)$, (16) and (18) that the kk Locus, depicted in Figure 2, shifts upward as a result of an increase in τ . Hence, in the extended model, the impact of changes in the cost of child quantity is amplified by the diluting effect on physical capital (the shift along the kk Locus as a result of the change in the ee Locus in addition to the change in the kk Locus). Furthermore, since the threshold level of education, e^T , declines with τ , a sufficient increase in the quantity cost can facilitate a demographic transition and release the economy from the poverty trap.

4 Concluding Remarks

This paper presents a model of fertility and child educational choice based on a reasonable and empirically supported assumption that individuals' productivity as teachers increases with their own human capital. The model offers an explanation for the persistence of poverty within and across countries, consistent with the negative relationship between fertility and education, and the cross-country relationship between education, fertility and economic growth. In contrast to the existing literature, the model generates multiple steady states based on the trade-off between child quality and quantity, without imposing any restrictive assumptions concerning preferences, the return to human capital, or any non-convexity in the production process.

The theory developed in this paper provides testable hypotheses as well as policy implications. An increase in the cost of quantity - the cost of a child regardless of the child's quality - induces a reallocation of resources to child quality. It reduces, therefore,

the level of the threshold below which individuals choose not to purchase any education for their offspring, and increases investment in education above this threshold. Hence, an increase in the quantity cost positively affects economic growth and could, furthermore, release an economy from the trap of poverty, setting the stage to a demographic transition and economic growth. This result is testable and bears policy implications. Variations in policies that reduce the quantity cost, such as tax discounts to large families, child allowances and subsidized day care and meals, can be exploited to uncover the effect of child cost on fertility and education decisions. According to the theory, these policies have a negative effect on income in the long run since they encourage households to increase fertility rates and reduce the quality investment and physical capital transfer to each child. Therefore, the policy implications, though not in harmony (at least in the short run) with a humanitarian approach, are straight-forward. Canceling, or even reversing, policies that reduce child quantity cost, would contribute to income per capita in the long run.²⁸

Furthermore, since public schooling can release the economy from the poverty trap, growth-encouraging policies include reallocating government or foreign aid resources from quantity-cost reduction measures to the finance of schooling. Note that providing education to a fraction of the population would have a positive effect on the economy. In contrast, according to Becker, Murphy and Tamura (1990), only policies that provide education to a large fraction of the population may have a positive long-run influence on the economy, since otherwise the returns to human capital will remain low. The finding that a temporary improvement in education opportunities could have a permanent effect on the distribution of skills was established in previous literature. The contribution of this paper, in this respect, is the linking of education to fertility, amplifying the economic consequences of the education policy.

The model offers explanations for cross-country output differences and for the phenomenon of club convergence. Consistent with the evidence, as shown for instance by Quah (1997), countries in the club of the rich converge to a high income per capita steady-state,

²⁸The justification to tax children follows from the pecuniary externality - population growth dilutes capital per capita. Of course, while negatively influencing income in the long run, the effect of some policies could be favorable to growth in the short run. For instance, day care can increase women's labor force participation.

whereas countries in the club of the poor converge to a low income level.²⁹ Consistent with the observation of Cohen (1996), the poor countries have failed to catch up with rich ones due to insufficient progress in education. According to the theory, the only difference between members of the two clubs is in the initial conditions. However, an important element in the explanation offered here, as well as in other club convergence theories, is missing. Since today's most developed economies were once poorer than most of today's poor economies, and since the dynamical system is constant over time and across economies, how did the developed economies pass the threshold level of income to form the club of the rich? The theory developed in this paper offers an insight to the resolution of this puzzle. Technological spillovers from the advanced economies to the poor ones may alter the quality-quantity price ratio, and hence, change the dynamical path, potentially generating a poverty trap. Furthermore, the model implies that capital flows from advanced economies to the poor induce a reallocation of resources, increasing fertility rates and reducing capital transfers to each child.

Finally, inferences from the extended model suggest that inequality has a negative effect on economic growth via the interaction between the rich and the poor. The relatively high fertility rates of the poor positively affect the return to physical capital. Therefore, the wealthy, who are more educated, reduce fertility and possibly education investment, and increase physical capital accumulation. On the other hand, due to the comparative advantage in child quantity, the increased physical capital accumulation by the wealthy turn the poor away from savings to increased fertility. The wealthy, therefore, specialize in accumulating wealth, while the poor specialize in high fertility rates, both negatively affecting the average level of human capital and output per capita in the economy.

²⁹See Azariadis (1996) and Galor (1996) for surveys of the theoretical and empirical literature and the summary by Durlauf (1996b).

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Figure 1. The evolution of education.

Offspring level education e_{t+1} is uniquely determined by parental education e_t .

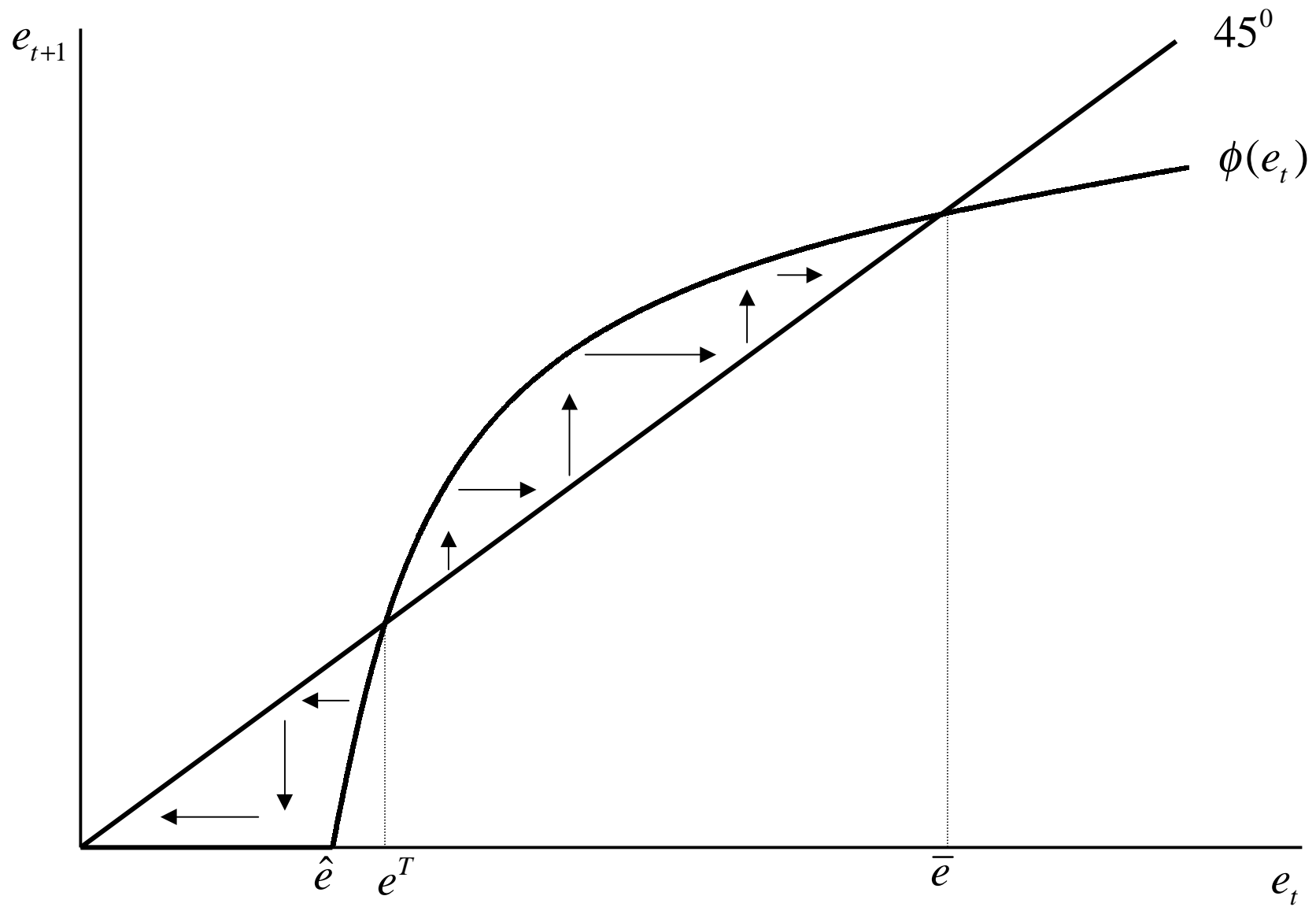


Figure 2. The dynamical system.

